# CHAPTER 1
Equations, Inequalities, and Mathematical Modeling

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CHAPTER 1
Equations, Inequalities, and Mathematical Modeling

Section 1.1 Graphs of Equations

- You should be able to use the point-plotting method of graphing.
- You should be able to find x- and y-intercepts.
  (a) To find the x-intercepts, let $y = 0$ and solve for $x$.
  (b) To find the y-intercepts, let $x = 0$ and solve for $y$.
- You should be able to test for symmetry.
  (a) To test for $x$-axis symmetry, replace $y$ with $-y$.
  (b) To test for $y$-axis symmetry, replace $x$ with $-x$.
  (c) To test for origin symmetry, replace $x$ with $-x$ and $y$ with $-y$.
- You should know the standard equation of a circle with center $(h, k)$ and radius $r$:
  $$(x - h)^2 + (y - k)^2 = r^2$$

Vocabulary Check

1. solution or solution point
2. graph
3. intercepts
4. $y$-axis
5. circle; $(h, k)$; $r$
6. point-plotting

1. $y = \sqrt{x + 4}$
   (a) $(0, 2)$: $2 = \sqrt{0 + 4}$
   \[2 = 2\]
   Yes, the point is on the graph.
   (b) $(5, 3)$: $3 = \sqrt{5 + 4}$
   \[3 = \sqrt{9}\]
   Yes, the point is on the graph.

2. $y = x^2 - 3x + 2$
   (a) $(2, 0)$: $(2)^2 - 3(2) + 2 = 0$
   \[4 - 6 + 2 = 0\]
   \[0 = 0\]
   Yes, the point is on the graph.
   (b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 = 8$
   \[4 + 6 + 2 = 8\]
   \[12 \neq 8\]
   No, the point is not on the graph.

3. $y = 4 - |x - 2|$
   (a) $(1, 5)$: $5 = 4 - |1 - 2|$
   \[5 = 4 - 1\]
   No, the point is not on the graph.
   (b) $(6, 0)$: $0 = 4 - |6 - 2|$
   \[0 = 4 - 4\]
   Yes, the point is on the graph.
4. \( y = \frac{1}{3}x^3 - 2x^2 \)

(a) \( (2, \ -\frac{16}{3}) \):
\[
\frac{1}{3}(2)^3 - 2(2)^2 = \frac{8}{3} - 16 = -\frac{40}{3}
\]
\[
\frac{8}{3} - 8 = -\frac{16}{3}
\]
\[
\frac{8}{3} - \frac{24}{3} = -\frac{16}{3}
\]
\[
-\frac{16}{3} = -\frac{16}{3}
\]
Yes, the point is on the graph.

(b) \( (-3, 9) \):
\[
\frac{1}{3}(-3)^3 - 2(-3)^2 = 9
\]
\[
\frac{1}{3}(-27) - 2(9) = 9
\]
\[
-9 - 18 = 9
\]
\[
-27 \neq 9
\]
No, the point is not on the graph.

5. \( y = -2x + 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(\frac{5}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(7)</td>
<td>(5)</td>
<td>(3)</td>
<td>(1)</td>
<td>(0)</td>
</tr>
<tr>
<td>((x, y))</td>
<td>((-1, 7))</td>
<td>((0, 5))</td>
<td>((1, 3))</td>
<td>((2, 1))</td>
<td>((\frac{5}{2}, 0))</td>
</tr>
</tbody>
</table>

6. \( y = \frac{3}{2}x - 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(0)</th>
<th>(1)</th>
<th>(\frac{4}{3})</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(-\frac{5}{3})</td>
<td>(-1)</td>
<td>(-\frac{1}{3})</td>
<td>(0)</td>
<td>(\frac{1}{2})</td>
</tr>
</tbody>
</table>

7. \( y = x^2 - 3x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(4)</td>
<td>(0)</td>
<td>(-2)</td>
<td>(-2)</td>
<td>(0)</td>
</tr>
<tr>
<td>((x, y))</td>
<td>((-1, 4))</td>
<td>((0, 0))</td>
<td>((1, -2))</td>
<td>((2, -2))</td>
<td>((3, 0))</td>
</tr>
</tbody>
</table>

8. \( 5 - x^2 \)

<table>
<thead>
<tr>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(4)</td>
<td>(5)</td>
<td>(4)</td>
<td>(1)</td>
</tr>
</tbody>
</table>

9. \( y = 16 - 4x^2 \)

x-intercepts: \( 0 = 16 - 4x^2 \)
\[
4x^2 = 16
\]
\[
x^2 = 4
\]
\[
x = \pm 2
\]
\( (-2, 0), (2, 0) \)

y-intercept: \( y = 16 - 4(0)^2 = 16 \)
\( (0, 16) \)
10. \(y = (x + 3)^2\)
   - \(x\)-intercept: \(0 = (x + 3)^2\)
     \[0 = x + 3\]
     \[x = -3\]
     \((-3, 0)\)
   - \(y\)-intercept: \(y = (0 + 3)^2\)
     \[y = 9\]
     \[(0, 9)\]

11. \(y = 2x^3 - 4x^2\)
   - \(x\)-intercepts: \(0 = 2x^3 - 4x^2\)
     \[0 = 2x^2(x - 2)\]
     \[x = 0\] or \(x = 2\)
     \[(0, 0), (2, 0)\]
   - \(y\)-intercept: \(y = 2(0)^3 - 4(0)^2\)
     \[y = 0\]
     \[(0, 0)\]

12. \(y^2 = x + 1\)
   - \(x\)-intercept: \(0 = x + 1\)
     \[x = -1\]
     \[(-1, 0)\]
   - \(y\)-intercepts: \(y^2 = 0 + 1\)
     \[y = \pm 1\]
     \[(0, 1), (0, -1)\]

13. \(y\)-axis symmetry

14. \(x^2 - y = 0\)
   \((-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow y\)-axis symmetry
   \(x^2 - (y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow \text{No } x\)-axis symmetry
   \((-x)^2 - (y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow \text{No origin symmetry}\)

15. Origin symmetry

16. \(x - y^2 = 0\)
   \(x - (-y)^2 = 0\)
   \(x - y^2 = 0\)

17. \(y^3\)
   - \(y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow \text{No } y\)-axis symmetry
   - \(-y = x^3 \Rightarrow y = -x^3 \Rightarrow \text{No } x\)-axis symmetry
   - \(-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow \text{Origin symmetry}\)

18. \(y = x^4 - x^2 + 3\)
   - \(y = (-x)^4 - (-x)^2 + 3\)
   - \(y = x^4 - x^2 + 3\)
   - \(y\)-axis symmetry
21. \( y = \frac{x}{x^2 + 1} \)

\[ \begin{align*}
  y = \frac{-x}{(-x)^2 + 1} & \Rightarrow y = \frac{-x}{x^2 + 1} \Rightarrow \text{No y-axis symmetry} \\
  -y = \frac{x}{x^2 + 1} & \Rightarrow y = \frac{-x}{x^2 + 1} \Rightarrow \text{No x-axis symmetry} \\
  -y = \frac{-x}{(-x)^2 + 1} & \Rightarrow -y = \frac{-x}{x^2 + 1} \Rightarrow y = \frac{x}{x^2 + 1} \Rightarrow \text{Origin symmetry}
\end{align*} \]

22. \( y = \sqrt{9 - x^2} \)

\[ \begin{align*}
  y = \sqrt{9 - (-x)^2} & \Rightarrow y = \sqrt{9 - x^2} \Rightarrow \text{y-axis symmetry}
\end{align*} \]

23. \( xy^2 + 10 = 0 \)

\( (-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \text{No y-axis symmetry} \)

\( x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow \text{x-axis symmetry} \)

\( (-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \text{No origin symmetry} \)

24. \( xy = 4 \)

\( (-x)(-y) = 4 \)

\( xy = 4 \)

Origin symmetry

25. \( y = -3x + 1 \)

\( x\)-intercept: \( \left( \frac{1}{3}, 0 \right) \)

\( y\)-intercept: \( (0, 1) \)

No axis or origin symmetry

26. \( y = 2x - 3 \)

\( x\)-intercept: \( \left( \frac{3}{2}, 0 \right) \)

\( y\)-intercept: \( (0, -3) \)

No symmetry

27. \( y = x^2 - 2x \)

Intercepts: \( (0, 0), (2, 0) \)

No axis or origin symmetry

\[ \begin{array}{c|cccc}
   x & -1 & 0 & 1 & 2 \\
   \hline
   y & 3 & 0 & -1 & 0
\end{array} \]

28. \( y = -x^2 - 2x \)

\( x\)-intercept: \( (-2, 0), (0, 0) \)

\( y\)-intercept: \( (0, 0) \)

No symmetry

29. \( y = x^3 + 3 \)

Intercepts: \( (0, 3), (\sqrt[3]{-3}, 0) \)

No axis or origin symmetry

\[ \begin{array}{c|cccc}
   x & -2 & -1 & 0 & 1 \\
   \hline
   y & -5 & 2 & 3 & 4
\end{array} \]
30. \( y = x^3 - 1 \)
   - \( x \)-intercept: \((1, 0)\)
   - \( y \)-intercept: \((0, -1)\)
   - No symmetry

31. \( y = \sqrt{x - 3} \)
   - Domain: \([3, \infty)\)
   - Intercept: \((3, 0)\)
   - No axis or origin symmetry

32. \( y = \sqrt{1 - x} \)
   - Domain: \(( -\infty, 1]\)
   - \( y \)-intercept: \((0, 1)\)
   - No symmetry

33. \( y = |x - 6| \)
   - Intercepts: \((0, 6), (6, 0)\)
   - No axis or origin symmetry

34. \( y = 1 - |x| \)
   - \( x \)-intercepts: \((\pm 1, 0)\)
   - \( y \)-intercept: \((0, 1)\)
   - \( y \)-axis symmetry

35. \( x = y^2 - 1 \)
   - Intercepts: \((0, -1), (0, 1), (-1, 0)\)
   - \( x \)-axis symmetry

36. \( x = y^2 - 5 \)
   - \( x \)-intercept: \((-5, 0)\)
   - \( y \)-intercept: \((0, \pm \sqrt{5})\)
   - \( x \)-axis symmetry

37. \( y = 3 - \frac{1}{2}x \)
   - Intercepts: \((6, 0), (0, 3)\)
Section 1.1  Graphs of Equations

38. \( y = \frac{2}{5} x - 1 \)

- Intercepts: \((0, -1), \left(\frac{5}{2}, 0\right)\)

39. \( y = x^2 - 4x + 3 \)

- Intercepts: \((3, 0), (1, 0), (0, 3)\)

40. \( y = x^2 + x - 2 \)

- Intercepts: \((-2, 0), (1, 0), (0, -2)\)

41. \( y = \frac{2x}{x - 1} \)

- Intercept: \((0, 0)\)

42. \( y = \frac{4}{x^2 + 1} \)

- Intercept: \((0, 4)\)

43. \( y = \sqrt[3]{x} \)

- Intercept: \((0, 0)\)

44. \( y = \sqrt[3]{x} + 1 \)

- Intercepts: \((-1, 0), (0, 1)\)

45. \( y = x\sqrt{x} + 6 \)

- Intercepts: \((0, 0), (-6, 0)\)

46. \( y = (6 - x)\sqrt{x} \)

- Intercepts: \((0, 0), (6, 0)\)

47. \( y = |x + 3| \)

- Intercepts: \((-3, 0), (0, 3)\)

48. \( y = 2 - |x| \)

- Intercepts: \((\pm 2, 0), (0, 2)\)

49. Center: \((0, 0)\); radius: 4

- Standard form:
  \[(x - 0)^2 + (y - 0)^2 = 4^2\]
  \[x^2 + y^2 = 16\]

50. \((x - 0)^2 + (y - 0)^2 = 5^2\)

- \(x^2 + y^2 = 25\)

51. Center: \((2, -1)\); radius: 4

- Standard form:
  \[(x - 2)^2 + (y + 1)^2 = 4^2\]
  \[(x - 2)^2 + (y + 1)^2 = 16\]

52. \((x - (-7))^2 + (y - (-4))^2 = 7^2\)

- \((x + 7)^2 + (y + 4)^2 = 49\)

53. Center: \((-1, 2)\); solution point: \((0, 0)\)

- \((x - (-1))^2 + (y - 2)^2 = r^2\)
- \((0 + 1)^2 + (0 - 2)^2 = r^2 \Rightarrow 5 = r^2\)
- Standard form: \((x + 1)^2 + (y - 2)^2 = 5\)

54. \(r = \sqrt{(3 - (-1))^2 + (-2 - 1)^2}\)

- \(= \sqrt{3^2 + (-3)^2} = \sqrt{18} = 5\)
- \((x - 3)^2 + (y - (-2))^2 = 5^2\)
  \[(x - 3)^2 + (y + 2)^2 = 25\]
55. Endpoints of a diameter: (0, 0), (6, 8)
   Center: \( \left( \frac{0 + 6}{2}, \frac{0 + 8}{2} \right) = (3, 4) \)
   \((x - 3)^2 + (y - 4)^2 = r^2\)
   \((0 - 3)^2 + (0 - 4)^2 = r^2 \Rightarrow 25 = r^2\)
   Standard form: \((x - 3)^2 + (y - 4)^2 = 25\)

56. \( r = \frac{1}{2}\sqrt{(-4 - 4)^2 + (-1 - 1)^2} \)
   \( = \frac{1}{2}\sqrt{(-8)^2 + (-2)^2} \)
   \( = \frac{1}{2}\sqrt{64 + 4} \)
   \( = \frac{1}{2}\sqrt{68} = \left(\frac{1}{2}\right)(2)\sqrt{17} = \sqrt{17} \)
   \( r^2 = \left(\frac{1}{2}\right)^2 \Rightarrow 25 = 68 \)
   Midpoint of diameter (center of circle):
   \( \left( \frac{-4 + 4}{2}, \frac{-1 + 1}{2} \right) = (0, 0) \)
   \((x - 0)^2 + (y - 0)^2 = \left(\sqrt{17}\right)^2 \)
   \( x^2 + y^2 = 17 \)

57. \( x^2 + y^2 = 25 \)
   Center: (0, 0), radius: 5

58. \( x^2 + y^2 = 16 \)
   Center: (0, 0), radius: 4

59. \( (x - 1)^2 + (y + 3)^2 = 9 \)
   Center: (1, -3), radius: 3

60. \( x^2 + (y - 1)^2 = 1 \)
   Center: (0, 1), radius: 1

61. \( (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{9}{4} \)
   Center: \( \left( \frac{1}{2}, \frac{1}{2} \right) \), radius: \( \frac{3}{2} \)

62. \( (x - 2)^2 + (y + 3)^2 = \frac{16}{9} \)
   Center: (2, -3), radius: \( \frac{4}{3} \)

63. \( y = 225,000 - 20,000t, \ 0 \leq t \leq 8 \)

64. \( y = 8100 - 929t, \ 0 \leq t \leq 6 \)
65. (a) \[ y = x \]

(b) \[ 2x + 2y = \frac{1040}{x} \]
\[ 2y = \frac{1040}{x} - 2x \]
\[ y = \frac{520}{x} - x \]
\[ A = xy = x \left( \frac{520}{x} - x \right) \]

(d) When \( x = y = 86.2 \) yards, the area is a maximum of \( 7511 \frac{1}{2} \) square yards.

(e) A regulation NFL playing field is 120 yards long and \( 53 \frac{1}{2} \) yards wide. The actual area is 6400 square yards.

66. (a) \[ y = x \]

(b) \( P = 360 \) meters so:
\[ 2x + 2y = 360 \]
\[ w = y = 180 - x \]
\[ A = lw = x(180 - x) \]

(d) \( x = 90 \) and \( y = 90 \)

A square will give the maximum area of 8100 square meters.

(e) The dimensions of a Major League Soccer field can vary between 110 and 120 yards in length and between 70 and 80 yards in width.

67. \( y = -0.0025t^2 + 0.574t + 44.25, \ 20 \leq t \leq 100 \)

(a) \( y \)

(c) For the year 1948, let \( t = 48: \ y \approx 66.0 \) years.

(d) For the year 2005, let \( t = 105: \ y \approx 77.0 \) years.

For the year 2010, let \( t = 110: \ y \approx 77.1 \) years.

(e) No. The graph reaches a maximum of \( y \approx 77.2 \) years when \( t \approx 114.8 \), or during the year 2014. After this time, the model has life expectancy decreasing, which is not realistic.

68. (a) \[ x \quad 5 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60 \quad 70 \quad 80 \quad 90 \quad 100 \]
\[ y \quad 430.43 \quad 107.33 \quad 26.56 \quad 11.60 \quad 6.36 \quad 3.94 \quad 2.62 \quad 1.83 \quad 1.31 \quad 0.96 \quad 0.71 \]

(b) \( y \)

(c) When \( x = 85.5 \),
\[ y = \frac{10.770}{85.5^2} - 0.37 = 1.10327. \]

(d) As the diameter of the wire increases, the resistance decreases.

69. False. A graph is symmetric with respect to the \( x \)-axis if, whenever \((x, y)\) is on the graph, \((x, -y)\) is also on the graph.

70. True. The graph can have no intercepts, one, two or many. For example, a circle centered at the origin has two \( y \)-intercepts. A circle of radius 1, centered at \( (7, 7) \), has no \( y \)-intercepts.
71. The viewing window is incorrect. Change the viewing window. Examples will vary. For example, \( y = x^2 + 20 \) will not appear in the standard window setting.

72. \( y = ax^2 + bx^3 \)
   
   \((a)\) \( y = a(-x)^2 + b(-x)^3 = ax^2 - bx^3 \)
   
   To be symmetric with respect to the y-axis, \( a \) can be any non-zero real number; \( b \) must be zero.
   
   \((b)\) \(-y = a(-x)^2 + b(-x)^3\)
   
   \(-y = ax^2 - bx^3\)
   
   \( y = -ax^2 + bx^3 \)
   
   To be symmetric with respect to the origin, \( a \) must be zero; \( b \) can be any non-zero real number.

73. \( 9x^5 + 4x^3 - 7 \)
   
   Terms: \( 9x^5, 4x^3, -7 \)

74. \(- (7 \times 7 \times 7) = -(7)^4 = -7^4 \)

75. \( \sqrt{18x} - \sqrt{2x} = 3\sqrt{2x} - \sqrt{2x} = 2\sqrt{2x} \)

76. \( \frac{55}{\sqrt{20} - 3} = \frac{55}{\sqrt{20} - 3} \cdot \frac{\sqrt{20} + 3}{\sqrt{20} + 3} \)
   
   \( = \frac{55(\sqrt{20} + 3)}{20 - 9} = \frac{55(\sqrt{20} + 3)}{11} \)
   
   \( = 5(\sqrt{20} + 3) = 5(2\sqrt{5} + 3) \)

77. \( \sqrt[3]{16} = |t|^{1/3} = 3\sqrt[3]{|t|} \)

78. \( \sqrt[6]{y} = (y^{1/2})^{1/3} = y^{1/6} = \sqrt[6]{y} \)

Section 1.2 Linear Equations in One Variable

- You should know how to solve linear equations.
  
  \( ax + b = 0, \ a \neq 0 \)

- An identity is an equation whose solution consists of every real number in its domain.

- To solve an equation you can:
  
  (a) Add or subtract the same quantity from both sides.
  
  (b) Multiply or divide both sides by the same nonzero quantity.
  
  (c) Remove all symbols of grouping and all fractions.
  
  (d) Combine like terms.
  
  (e) Interchange the two sides.

- Check the answer!

- A “solution” that does not satisfy the original equation is called an extraneous solution.

- Be able to find intercepts algebraically.

Vocabulary Check

1. equation
2. solve
3. identities; conditional
4. \( ax + b = 0 \)
5. extraneous
1. $5x - 3 = 3x + 5$
   (a) $5(0) - 3 \neq 3(0) + 5$
       $-3 \neq 5$
       $x = 0$ is not a solution.
   (c) $5(4) - 3 \neq 3(4) + 5$
       $17 = 17$
       $x = 4$ is a solution.

2. $7 - 3x = 5x - 17$
   (a) $x = -3$
       $7 - 3(-3) \neq 5(-3) - 17$
       $7 + 9 \neq -15 - 17$
       $16 \neq -32$
       $x = -3$ is not a solution.
   (c) $x = 8$
       $7 - 3(8) \neq 5(8) - 17$
       $7 - 24 \neq 40 - 17$
       $-17 \neq 23$
       $x = 8$ is not a solution.

3. $3x^2 + 2x - 5 = 2x^2 - 2$
   (a) $3(-3)^2 + 2(-3) - 5 \neq 2(-3)^2 - 2$
       $27 - 6 - 5 \neq 18 - 2$
       $16 = 16$
       $x = -3$ is a solution.
   (c) $3(4)^2 + 2(4) - 5 \neq 2(4)^2 - 2$
       $48 + 8 - 5 \neq 32 - 2$
       $51 \neq 30$
       $x = 4$ is not a solution.

4. $5x^3 + 2x - 3 = 4x^3 + 2x - 11$
   (a) $x = 2$
       $5(2)^3 + 2(2) - 3 \neq 4(2)^3 + 2(2) - 11$
       $5 \cdot 8 + 4 - 3 \neq 4(8) + 4 - 11$
       $40 + 4 - 3 \neq 32 + 4 - 11$
       $41 \neq 25$
       $x = 2$ is not a solution.
   (c) $x = 0$
       $5(0)^3 + 2(0) - 3 \neq 4(0)^3 + 2(0) - 11$
       $0 + 0 - 3 \neq 0 + 0 - 11$
       $-3 \neq -11$
       $x = 0$ is not a solution.

(b) $5(-5) - 3 \neq 3(-5) + 5$
    $-28 \neq -10$
    $x = -5$ is not a solution.

(d) $5(10) - 3 \neq 3(10) + 5$
    $47 \neq 35$
    $x = 10$ is not a solution.
5. \( \frac{5}{2} - \frac{4}{x} = 3 \)

(a) \( \frac{5}{2(-1/2)} - \frac{4}{(-1/2)} \equiv 3 \)
\[-5 + 8 \equiv 3 \]
\[3 = 3 \]
\[x = -\frac{1}{2} \text{ is a solution.} \]

(b) \( \frac{5}{2(4)} - \frac{4}{4} \equiv 3 \)
\[\frac{5}{8} - 1 \equiv 3 \]
\[\frac{3}{8} \not\equiv 3 \]
\[x = 4 \text{ is not a solution.} \]

(c) \( \frac{5}{2(0)} - \frac{4}{0} \) is undefined.
\[x = 0 \text{ is not a solution.} \]

(d) \( \frac{5}{2(1/4)} - \frac{4}{1/4} \equiv 3 \)
\[10 - 16 \equiv 3 \]
\[\not\equiv 3 \]
\[x = \frac{1}{4} \text{ is not a solution.} \]

6. \( 3 + \frac{1}{x + 2} = 4 \)

(a) \( x = -1 \)
\[\frac{3}{3} + \frac{1}{-1 + 2} \equiv 4 \]
\[\frac{3}{1} + \frac{1}{1} \equiv 4 \]
\[3 + 1 \equiv 4 \]
\[4 = 4 \]
\[x = -1 \text{ is a solution.} \]

(b) \( x = -2 \)
\[\frac{3}{3} + \frac{1}{-2 + 2} \equiv 4 \]
\[\frac{3}{0} + \frac{1}{0} \equiv 4 \]
\[\not\equiv 4 \]
\[x = -2 \text{ is not a solution.} \]

(c) \( x = 0 \)
\[\frac{3}{3} + \frac{1}{0 + 2} \equiv 4 \]
\[\frac{3}{2} + \frac{1}{2} \equiv 4 \]
\[\frac{3}{2} 
ot\equiv 4 \]
\[x = 0 \text{ is not a solution.} \]

(d) \( x = 5 \)
\[\frac{3}{3} + \frac{1}{5 + 2} \equiv 4 \]
\[\frac{3}{7} + \frac{1}{7} \equiv 4 \]
\[\frac{3}{7} 
ot\equiv 4 \]
\[x = 5 \text{ is not a solution.} \]

7. \( \sqrt{3x - 2} = 4 \)

(a) \( \sqrt{3(3) - 2} \equiv 4 \)
\[\sqrt{7} \not\equiv 4 \]
\[x = 3 \text{ is not a solution.} \]

(b) \( \sqrt{3(2) - 2} \equiv 4 \)
\[\sqrt{4} \equiv 4 \]
\[x = 2 \text{ is not a solution.} \]

(c) \( \sqrt{3(9) - 2} \equiv 4 \)
\[\sqrt{25} \not\equiv 4 \]
\[x = 9 \text{ is not a solution.} \]

(d) \( \sqrt{3(-6) - 2} \equiv 4 \)
\[\sqrt{-20} \not\equiv 4 \]
\[x = -6 \text{ is not a solution.} \]

8. \( \sqrt{x - 8} = 3 \)

(a) \( x = 2 \)
\[\sqrt{2 - 8} \equiv 3 \]
\[\sqrt{-6} \not\equiv 3 \]
\[x = 2 \text{ is not a solution.} \]

(b) \( x = -5 \)
\[\sqrt{-5 - 8} \equiv 3 \]
\[\sqrt{-13} \not\equiv 3 \]
\[x = -5 \text{ is not a solution.} \]

(c) \( x = 35 \)
\[\sqrt{35 - 8} \equiv 3 \]
\[\sqrt{27} \equiv 3 \]
\[0 \not\equiv 3 \]
\[x = 3 \text{ is not a solution.} \]

(d) \( x = 8 \)
\[\sqrt{8 - 8} \equiv 3 \]
\[\sqrt{0} \equiv 3 \]
\[0 \not\equiv 3 \]
\[x = 8 \text{ is not a solution.} \]

9. \( 6x^2 - 11x - 35 = 0 \)

(a) \( 6\left(-\frac{5}{3}\right)^2 - 11\left(-\frac{5}{3}\right) - 35 \equiv 0 \)
\[\frac{50}{3} + \frac{55}{3} - \frac{105}{3} \not\equiv 0 \]
\[0 = 0 \]
\[x = -\frac{5}{3} \text{ is a solution.} \]

(b) \( 6\left(-\frac{7}{2}\right)^2 - 11\left(-\frac{7}{2}\right) - 35 \equiv 0 \)
\[\frac{24}{9} + \frac{154}{9} - \frac{1715}{9} \not\equiv 0 \]
\[\frac{-1537}{9} \not= 0 \]
\[x = -\frac{7}{2} \text{ is not a solution.} \]

(c) \( 6\left(\frac{7}{2}\right)^2 - 11\left(\frac{7}{2}\right) - 35 \equiv 0 \)
\[\frac{147}{2} - \frac{77}{2} - \frac{70}{2} \not= 0 \]
\[0 = 0 \]
\[x = \frac{7}{2} \text{ is a solution.} \]

(d) \( 6\left(\frac{5}{3}\right)^2 - 11\left(\frac{5}{3}\right) - 35 \equiv 0 \)
\[\frac{50}{3} - \frac{55}{3} - \frac{105}{3} \not= 0 \]
\[\frac{-110}{3} \not= 0 \]
\[x = \frac{5}{3} \text{ is not a solution.} \]
10. \(10x^2 + 21x - 10 = 0\)

(a) \(x = \frac{2}{5}\)
\[
\begin{align*}
10(\frac{2}{5})^2 + 21(\frac{2}{5}) - 10 &= 0 \\
10(\frac{4}{25}) + 21(\frac{2}{5}) - 10 &= 0 \\
\frac{4}{5} + \frac{42}{5} - \frac{50}{5} &= 0 \\
0 &= 0
\end{align*}
\]
\(x = \frac{2}{5}\) is a solution.

(c) \(x = -\frac{1}{4}\)
\[
\begin{align*}
10\left(-\frac{1}{4}\right)^2 + 21\left(-\frac{1}{4}\right) - 10 &= 0 \\
10\left(-\frac{1}{16}\right) - 7 - 10 &= 0 \\
-\frac{10}{16} - \frac{112}{16} &= 0 \\
-\frac{122}{16} &= 0
\end{align*}
\]
x = -\(\frac{1}{4}\) is not a solution.

(b) \(x = -\frac{5}{2}\)
\[
\begin{align*}
10\left(-\frac{5}{2}\right)^2 + 21\left(-\frac{5}{2}\right) - 10 &= 0 \\
10\left(\frac{25}{4}\right) - \frac{105}{2} &= 0 \\
\frac{250}{4} - \frac{105}{2} &= 0 \\
0 &= 0
\end{align*}
\]
x = -\(\frac{5}{2}\) is a solution.

(d) \(x = -2\)
\[
\begin{align*}
10(-2)^2 + 21(-2) - 10 &= 0 \\
10(4) - 42 - 10 &= 0 \\
40 - 42 - 10 &= 0 \\
-12 &= 0
\end{align*}
\]
x = -2 is not a solution.

11. \(2(x - 1) = 2x - 2\) is an identity by the Distributive Property. It is true for all real values of \(x\).

13. -6(\(x - 3\)) + 5 = -2x + 10 is conditional. There are real values of \(x\) for which the equation is not true.

15. \(4(x + 1) - 2x = 4x + 4 - 2x = 2x + 4 = 2(x + 2)\)
This is an identity by simplification. It is true for all real values of \(x\).

17. \((x - 4)^2 - 11 = x^2 - 8x + 16 - 11 = x^2 - 8x + 5\)
Thus, \(x^2 - 8x + 5 = (x - 4)^2 - 11\) is an identity by simplification. It is true for all real values of \(x\).

19. \(\frac{3 + 1}{x + 1} = \frac{4x}{x + 1}\) is conditional. There are real values of \(x\) for which the equation is not true.

21. \(4x + 32 = 83\)
Original equation
\[
\begin{align*}
4x + 32 - 32 &= 83 - 32 \\
4x &= 51 \\
\frac{4x}{4} &= \frac{51}{4} \\
x &= \frac{51}{4}
\end{align*}
\]

22. \(3(x - 4) + 10 = 7\)
Original equation
\[
\begin{align*}
3x - 12 + 10 &= 7 \\
3x - 2 &= 7 \\
3x &= 9 \\
\frac{3x}{3} &= \frac{9}{3} \\
x &= 3
\end{align*}
\]
23. \( x + 11 = 15 \)  
\[ x + 11 - 11 = 15 - 11 \]  
\[ x = 4 \]

24. \( 7 - x = 19 \)  
\[ 7 - x + x = 19 + x \]  
\[ 7 = 19 + x \]  
\[ 7 - 19 = 19 + x - 19 \]  
\[ -12 = x \]

25. \( 7 - 2x = 25 \)  
\[ 7 - 7 - 2x = 25 - 7 \]  
\[ -2x = 18 \]  
\[ \frac{-2x}{-2} = \frac{18}{-2} \]  
\[ x = -9 \]

26. \( 7x + 2 = 23 \)  
\[ 7x + 2 - 2 = 23 - 2 \]  
\[ 7x = 21 \]  
\[ \frac{7x}{7} = \frac{21}{7} \]  
\[ x = 3 \]

27. \( 8x - 5 = 3x + 20 \)  
\[ 8x - 3x - 5 = 3x - 3x + 20 \]  
\[ 5x - 5 = 20 \]  
\[ 5x - 5 + 5 = 20 + 5 \]  
\[ 5x = 25 \]  
\[ \frac{5x}{5} = \frac{25}{5} \]  
\[ x = 5 \]

28. \( 7x + 3 = 3x - 17 \)  
\[ 7x + 3 - 3 - 3x = 3x - 17 - 3 - 3x \]  
\[ 4x = -20 \]  
\[ x = -5 \]

29. \( 2(x + 5) - 7 = 3(x - 2) \)  
\[ 2x + 10 - 7 = 3x - 6 \]  
\[ 2x + 3 = 3x - 6 \]  
\[ 2x - 3x + 3 = 3x - 3x - 6 \]  
\[ -x + 3 - 3 = -6 - 3 \]  
\[ -x = -9 \]  
\[ x = 9 \]

30. \( 3(x + 3) = 5(1 - x) - 1 \)  
\[ 3x + 9 = 5 - 5x - 1 \]  
\[ 3x + 9 = 4 - 5x \]  
\[ 3x + 9 + 5x - 9 = 4 - 5x + 5x - 9 \]  
\[ 8x = -5 \]  
\[ x = -\frac{5}{8} \]

31. \( x - 3(2x + 3) = 8 - 5x \)  
\[ x - 6x - 9 = 8 - 5x \]  
\[ -5x - 9 = 8 - 5x \]  
\[ -5x + 5x - 9 = 8 - 5x + 5x \]  
\[ -9 \neq 8 \]  
No solution

32. \( 9x - 10 = 5x + 2(2x - 5) \)  
\[ 9x - 10 = 5x + 4x - 10 \]  
\[ 9x - 10 = 9x - 10 \]  
The solution is the set of all real numbers.

33. \( \frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2} \)  
\[ 4\left(\frac{5x}{4} + \frac{1}{2}\right) = 4\left(x - \frac{1}{2}\right) \]  
\[ 4\left(\frac{5x}{4}\right) + 4\left(\frac{1}{2}\right) = 4(x) - 4\left(\frac{1}{2}\right) \]  
\[ 5x + 2 = 4x - 2 \]  
\[ x = -4 \]

34. \( \frac{x}{5} - \frac{x}{2} = 3 + \frac{3x}{10} \)  
\[ 10\left(\frac{x}{5} - \frac{x}{2}\right) = 10\left(3 + \frac{3x}{10}\right) \]  
\[ 2x - 5x = 30 + 3x \]  
\[ -6x = 30 \]  
\[ x = -5 \]

35. \( \frac{3}{2}(z + 5) - \frac{1}{4}(z + 24) = 0 \)  
\[ 4\left[\frac{3}{2}(z + 5) - \frac{1}{4}(z + 24)\right] = 4(0) \]  
\[ 4\left(\frac{3}{2}\right)(z + 5) - \frac{1}{4}(z + 24) = 4(0) \]  
\[ 6(z + 5) - (z + 24) = 0 \]  
\[ 6z + 30 - z - 24 = 0 \]  
\[ 5z = -6 \]  
\[ z = -\frac{6}{5} \]

36. \( \frac{3x}{2} + \frac{1}{4}(x - 2) = 10 \)  
\[ 4\left(\frac{3x}{2} + \frac{1}{4}(x - 2)\right) = 4(10) \]  
\[ 4\left(\frac{3x}{2}\right) + 4\left(\frac{1}{4}\right)(x - 2) = 4(10) \]  
\[ 6x + (x - 2) = 40 \]  
\[ 7x - 2 = 40 \]  
\[ 7x = 42 \]  
\[ x = 6 \]
37. \[0.25x + 0.75(10 - x) = 3\]
\[0.25x + 7.5 - 0.75x = 3\]
\[-0.50x + 7.5 = 3\]
\[-0.50x = -4.5\]
\[x = 9\]

39. \[3(x - 1) = 4\]
\[3x - 3 = 4\]
\[x = \frac{7}{3}\]

The second way is easier since you are not working with fractions until the end of the solution.

41. \[\frac{1}{3}(x + 2) = 5\]
\[\frac{1}{3}(x + 2) = 3(5)\]
\[x + 2 = 15\]
\[x = 13\]

The first way is easier. The fraction is eliminated in the first step.

43. \[x + 8 = 2(x - 2) - x\]
\[x + 8 = 2x - 4 - x\]
\[x + 8 = x - 4\]
\[8 = -4\]

Contradiction; no solution

45. \[\frac{100 - 4x}{3} = \frac{5x + 6}{4} + 6\]
\[12\left(\frac{100 - 4x}{3}\right) = 12\left(\frac{5x + 6}{4}\right) + 12(6)\]
\[4(100 - 4x) = 3(5x + 6) + 72\]
\[400 - 16x = 15x + 18 + 72\]
\[-31x = -310\]
\[x = 10\]
47. \[ \frac{5x - 4}{5x + 4} = \frac{2}{3} \]

\[ 3(5x - 4) = 2(5x + 4) \]

\[ 15x - 12 = 10x + 8 \]

\[ 5x - 20 = 0 \]

\[ x = 4 \]

48. \[ \frac{10x + 3}{5x + 6} = \frac{1}{2} \]

\[ 2(10x + 3) = 1(5x + 6) \]

\[ 20x + 6 = 5x + 6 \]

\[ 15x = 0 \]

\[ x = 0 \]

49. \[ 10 - \frac{13}{x} = 4 + \frac{5}{x} \]

\[ \frac{10x - 13}{x} = \frac{4x + 5}{x} \]

\[ 10x - 13 = 4x + 5 \]

\[ 6x = 18 \]

\[ x = 3 \]

50. \[ \frac{15x}{x} - 4 = \frac{6x}{x} + 3 \]

\[ \frac{15}{x} - 6 = 7 \]

\[ \frac{9}{x} = 7 \]

\[ 9 = 7x \]

\[ \frac{9}{7} = x \]

51. \[ 3 = 2 + \frac{2}{z + 2} \]

\[ 3z + 2 = (2 + \frac{2}{z + 2})(z + 2) \]

\[ 3z + 6 = 2z + 4 + 2 \]

\[ z = 0 \]

52. \[ \frac{1}{x} + \frac{2}{x - 5} = 0 \]

Multiply both sides by \(x(x - 5)\).

\[ 1(x - 5) + 2x = 0 \]

\[ 3x - 5 = 0 \]

\[ 3x = 5 \]

\[ x = \frac{5}{3} \]

53. \[ \frac{x}{x + 4} + \frac{4}{x + 4} + 2 = 0 \]

Multiply both sides by \(\frac{x + 4}{x + 4}\).

\[ \frac{x + 4}{x + 4} + 2 = 0 \]

\[ 1 + 2 = 0 \]

\[ 3 \neq 0 \]

Contradiction; no solution

54. \[ \frac{7}{2x + 1} - \frac{8x}{2x - 1} = -4 \]

Multiply both sides by \((2x + 1)(2x - 1)\).

\[ 7(2x - 1) - 8x(2x + 1) = -4(2x + 1)(2x - 1) \]

\[ 14x - 7 - 16x^2 - 8x = -4(2x^2 - 1) \]

\[ 6x = 11 \]

\[ x = \frac{11}{6} \]

55. \[ \frac{2}{(x - 4)(x - 2)} = \frac{1}{x - 4} + \frac{2}{x - 2} \]

Multiply both sides by \((x - 4)(x - 2)\).

\[ 2 = 1(x - 2) + 2(x - 4) \]

\[ 2 = x - 2 + 2x - 8 \]

\[ 2 = 3x - 10 \]

\[ 12 = 3x \]

\[ 4 = x \]

A check reveals that \(x = 4\) is an extraneous solution—it makes the denominator zero. There is no real solution.
60. \[ \frac{6}{x} - \frac{2}{x + 3} = \frac{3(x + 5)}{x(x + 3)} \] Multiply both sides by \(x(x + 3)\).

\[ 6(x + 3) - 2x = 3(x + 5) \]
\[ 6x + 18 - 2x = 3x + 15 \]
\[ 4x + 18 = 3x + 15 \]
\[ x = -3 \]

\text{Check:} \quad \frac{6}{-3} - \frac{2}{-3 + 3} = \frac{3(-3 + 5)}{-3(-3 + 3)}
\[ -2 - \frac{2}{0} = \frac{-6}{-3(0)} \]

Division by zero is undefined. Thus, \(x = -3\) is not a solution, and the original equation has no solution.
61. \((x + 2)^2 + 5 = (x + 3)^2\)
\[x^2 + 4x + 4 + 5 = x^2 + 6x + 9\]
\[4x + 9 = 6x + 9\]
\[-2x = 0\]
\[x = 0\]

62. \((x + 1)^2 + 2(x - 2) = (x + 1)(x - 2)\)
\[x^2 + 2x + 1 + 2x - 4 = x^2 - x - 2\]
\[5x = 1\]
\[x = \frac{1}{5}\]

63. \((x + 2)^2 - x^2 = 4(x + 1)\)
\[x^2 + 4x + 4 - x^2 = 4x + 4\]
\[4 = 4\]

The equation is an identity; every real number is a solution.

64. \((2x + 1)^2 = 4(x^2 + x + 1)\)
\[4x^2 + 4x + 1 = 4x^2 + 4x + 4\]
\[1 = 4\]

This is a contradiction. Thus, the equation has no solution.

65. \(y = 2(x - 1) - 4\)
\[0 = 2(x - 1) - 4\]
\[0 = 2x - 2 - 4\]
\[0 = 2x - 6\]
\[6 = 2x\]
\[3 = x\]
\[x = 3\]

The \(x\)-intercept is at 3. The solution to \(0 = 2(x - 1) - 4\) and the \(x\)-intercept of \(y = 2(x - 1) - 4\) are the same. They are both \(x = 3\). The \(x\)-intercept is (3, 0).

66. \(y = \frac{4}{3}x + 2\)
\[0 = \frac{4}{3}x + 2\]
\[-\frac{4}{3}x = 2\]
\[\left(-\frac{3}{2}\right)\left(-\frac{4}{3}x\right) = \left(-\frac{3}{2}\right)\left(2\right)\]
\[x = -\frac{3}{2}\]

Intercept: \((-\frac{3}{2}, 0)\)

The solution to \(0 = \frac{4}{3}x + 2\) is the same as the \(x\)-intercept of \(y = \frac{4}{3}x + 2\). They are both \(x = -\frac{3}{2}\).

67. \(y = 20 - (3x - 10)\)
\[0 = 20 - (3x - 10)\]
\[0 = 20 - 3x + 10\]
\[0 = 30 - 3x\]
\[3x = 30\]
\[x = 10\]

The \(x\)-intercept is at 10. The solution to \(0 = 20 - (3x - 10)\) and the \(x\)-intercept of \(y = 20 - (3x - 10)\) are the same. They are both \(x = 10\). The \(x\)-intercept is (10, 0).

68. \(y = 10 + 2(x - 2)\)
\[0 = 10 + 2(x - 2)\]
\[0 = 10 + 2x - 4\]
\[0 = 6 + 2x\]
\[-2x = 6\]
\[x = -3\]

Intercept: \((-3, 0)\)

The solution to \(0 = 10 + 2(x - 2)\) is the same as the \(x\)-intercept of \(y = 10 + 2(x - 2)\). They are both \(x = -3\).

69. \(y = -38 + 5(9 - x)\)
\[0 = -38 + 5(9 - x)\]
\[0 = -38 + 45 - 5x\]
\[0 = 7 - 5x\]
\[5x = 7\]
\[x = \frac{7}{5}\]

The \(x\)-intercept is \(\frac{7}{5}\). The solution to \(0 = -38 + 5(9 - x)\) and the \(x\)-intercept of \(y = -38 + 5(9 - x)\) are the same. They are both \(x = \frac{7}{5}\). The \(x\)-intercept is \((\frac{7}{5}, 0)\).

70. \(y = 6x - 6\left(\frac{16}{11} + x\right)\)
\[0 = 6x - 6\left(\frac{16}{11} + x\right)\]
\[0 = 6x - \frac{96}{11} - 6x\]
\[0 = -\frac{96}{11}\]

There is no \(x\)-intercept.
71. \( y = 12 - 5x \)
   - x-intercept: \( 0 = 12 - 5x \implies 5x = 12 \implies x = \frac{12}{5} \)
   - y-intercept: \( y = 12 - 5(0) \implies y = 12 \)
   - The x-intercept is \( \left( \frac{12}{5}, 0 \right) \) and the y-intercept is \( (0, 12) \).

72. \( y = 16 - 3x \quad y = 16 - 3x \)
   - \( 0 = 16 - 3x \quad y = 16 - 3(0) \)
   - \(-16 = -3x \quad y = 16 \)
   - \( \frac{-16}{-3} = x \)
   - The x-intercept is \( \left( \frac{16}{3}, 0 \right) \); the y-intercept is \( (0, 16) \).

73. \( y = -3(2x + 1) \)
   - x-intercept: \( 0 = -3(2x + 1) \implies 0 = 2x + 1 \implies x = -\frac{1}{2} \)
   - y-intercept: \( y = -3(2(0) + 1) \implies y = -3 \)
   - The x-intercept is \( \left( -\frac{1}{2}, 0 \right) \) and the y-intercept is \( (0, -3) \).

74. \( y = 5 - (6 - x) \quad y = 5 - (6 - x) \)
   - \( 0 = 5 - (6 - x) \quad y = 5 - (6 - 0) \)
   - \( 0 = -1 + x \quad y = -1 \)
   - \( 1 = x \)
   - The x-intercept is \( (1, 0) \); the y-intercept is \( (0, -1) \).

75. \( 2x + 3y = 10 \)
   - x-intercept: \( 2x + 3(0) = 10 \implies 2x = 10 \implies x = 5 \)
   - y-intercept: \( 2(0) + 3y = 10 \implies 3y = 10 \implies y = \frac{10}{3} \)
   - The x-intercept is \( (5, 0) \) and the y-intercept is \( (0, \frac{10}{3}) \).

76. \( 4x - 5y = 12 \quad 4x - 5y = 12 \)
   - \( 4x - 5(0) = 12 \quad 4(0) - 5y = 12 \)
   - \( 4x = 12 \quad -5y = 12 \)
   - \( x = 3 \quad y = -\frac{12}{5} \)
   - The x-intercept is \( (3, 0) \); the y-intercept is \( (0, -\frac{12}{5}) \).

77. \( \frac{2x}{5} + 8 - 3y = 0 \implies 2x + 40 - 15y = 0 \)
   - Multiply both sides by 5.
   - x-intercept: \( 2x + 40 - 15(0) = 0 \implies 2x + 40 = 0 \implies x = -20 \)
   - y-intercept: \( 2(0) + 40 - 15y = 0 \implies 40 - 15y = 0 \implies y = \frac{40}{15} = \frac{8}{3} \)
   - The x-intercept is \( (-20, 0) \) and the y-intercept is \( \left( 0, \frac{8}{3} \right) \).

78. \( \frac{8x}{3} + 5 - 2y = 0 \quad \frac{8x}{3} + 5 - 2y = 0 \)
   - \( \frac{8x}{3} + 5 - 2(0) = 0 \quad \frac{8(0)}{3} + 5 - 2y = 0 \)
   - \( \frac{8x}{3} = -5 \quad -2y = -5 \)
   - \( x = -\frac{15}{8} \quad y = \frac{5}{2} \)
   - The x-intercept is \( \left( -\frac{15}{8}, 0 \right) \); the y-intercept is \( \left( 0, \frac{5}{2} \right) \).

79. \( 4y - 0.75x + 1.2 = 0 \)
   - x-intercept: \( 4(0) - 0.75x + 1.2 = 0 \implies -0.75x + 1.2 = 0 \implies x = \frac{1.2}{0.75} = 1.6 \)
   - y-intercept: \( 4y - 0.75(0) + 1.2 = 0 \implies 4y + 1.2 = 0 \implies y = \frac{-1.2}{4} = -0.3 \)
   - The x-intercept is \( (1.6, 0) \) and the y-intercept is \( (0, -0.3) \).
80. \(3y + 2.5x - 3.4 = 0\) 
\[3(0) + 2.5x - 3.4 = 0\] 
\[2.5x = 3.4\] 
\[x = \frac{3.4}{2.5}\] 
\[x = 1.36\]

The \(x\)-intercept is \((1.36, 0)\); the \(y\)-intercept is \((0, 1.13)\).

81. \(4(x + 1) - ax = x + 5\) 
\[4x + 4 - ax = x + 5\] 
\[3x - ax = 1\] 
\[x(3 - a) = 1\] 
\[x = \frac{1}{3 - a}, a \neq 3\]

82. \(4 - 2(x - 2b) = ax + 3\) 
\[4 - 2x + 4b = ax + 3\] 
\[1 + 4b = 2x + ax\] 
\[1 + 4b = x(2 + a)\] 
\[1 + 4b = x, a \neq -2\]

83. \(6x + ax = 2x + 5\) 
\[4x + ax = 5\] 
\[x(4 + a) = 5\] 
\[x = \frac{5}{4 + a}, a \neq -4\]

84. \(5 + ax = 12 - bx\) 
\[ax + bx = 7\] 
\[x(a + b) = 7\] 
\[x = \frac{7}{a + b}, a \neq -b\]

85. \(19x + \frac{1}{2}ax = x + 9\) 
\[18x + \frac{1}{2}ax = 9\] 
Multiply both sides by 2.
\[36x + ax = 18\] 
\[x(36 + a) = 18\] 
\[x = \frac{18}{36 + a}, a \neq -36\]

86. \(-5(3x - 6b) + 12 = 8 + 3ax\) 
\[-15x + 30b + 12 = 8 + 3ax\] 
\[x(15 + 3a) = 30b + 4\] 
\[x = \frac{30b + 4}{3a + 15}, a \neq -5\]

87. \(-2ax + 6(x + 3) = -4x + 1\) 
\[-2ax + 6x + 18 = -4x + 1\] 
\[-2ax + 10x + 18 = 1\] 
\[-2ax + 10x = -17\] 
\[x(-2a + 10) = -17\] 
\[x = \frac{-17}{-2a + 10} = \frac{-17}{10 - 2a}, a \neq 5\]

88. \(\frac{4}{5}x - ax = 2\left(\frac{2}{5}x - 1\right) + 10\) 
\[\frac{4}{5}x - ax = \frac{4}{5}x - 2 + 10\] 
\[ax = -8\] 
\[x = \frac{-8}{a}, a \neq 0\]

89. \(0.275x + 0.725(500 - x) = 300\) 
\[0.275x + 362.5 - 0.725x = 300\] 
\[-0.45x = -62.5\] 
\[x = \frac{62.5}{0.45}\] 
\[\approx 138.889\]

90. \(2.763 - 4.5(2.1x - 5.1432) = 6.32x + 5\) 
\[2.763 - 9.45x + 23.1444 = 6.32x + 5\] 
\[20.9074 = 15.77x\] 
\[1.326 \approx x\]
91. \[
\frac{2}{7.398} - \frac{4.405}{x} = \frac{1}{x}
\]
Multiply both sides by 7.398x.

\[
2x - (4.405)(7.398) = 7.398
\]
\[
2x = (4.405)(7.398) + 7.398
\]
\[
2x = (5.405)(7.398)
\]
\[
x = \frac{(5.405)(7.398)}{2} \approx 19.993
\]

93. \[471 = 2\pi(25) + 2\pi(5h)\]
\[471 = 50\pi + 10\pi h\]
\[471 - 50\pi = 10\pi h\]
\[
h = \frac{471 - 50\pi}{10\pi} = \frac{471 - 50(3.14)}{10(3.14)} = 10
\]
\[h = 10 \text{ feet}\]

95. (a) Female: \[y = 0.432x - 10.44\]
For \[y = 16\]:
\[16 = 0.432x - 10.44\]
\[26.44 = 0.432x\]
\[
x = \frac{26.44}{0.432} \approx 61.2 \text{ inches}\]

(b) Male: \[y = 0.449x - 12.15\]
For \[y = 19\]:
\[19 = 0.449x - 12.15\]
\[31.15 = 0.449x\]
\[69.4 \approx x\]

Yes, it is likely that both bones came from the same person because the estimated height of a male with a 19-inch thigh bone is 69.4 inches.

(d) \[0.432x - 10.44 = 0.449x - 12.15\]
\[1.71 = 0.017x\]
\[x \approx 100.59 \text{ inches}\]

It is unlikely that a female would be over 8 feet tall, so if a femur of this length was found, it most likely belonged to a very tall male.

96. (a) \[T = E + S\]
\[T = E + \left(10,000 - \frac{1}{2}E\right)\]
\[T = 10,000 + \frac{1}{2}E, 0 \leq E \leq 20,000\]

(b) \[S = 10,000 - \frac{1}{2}E\]
For \[S = 6600\]:
\[6600 = 10,000 - \frac{1}{2}E\]
\[\frac{1}{2}E = 3400\]
\[E = 6800\]

The earned income \(E\) is $6800.

(c) \[13,800 = 10,000 + \frac{1}{2}E\]
\[3800 = \frac{1}{2}E\]
\[7600 = E\]

Earned income: $7600

(d) \[T = 10,000 + \frac{1}{2}E\]
For \[T = 12,500\]:
\[12,500 = 10,000 + \frac{1}{2}E\]
\[2500 = \frac{1}{2}E\]
\[5000 = E\]
\[S = 10,000 - \frac{1}{2}E = 10,000 - \frac{1}{2}(5000)\]
\[= 10,000 - 2500 = 7500\]

The subsidy is $7500.
97. \( y = 1.64t + 36.8, \ -1 \leq t \leq 12 \)

(a) The intercept is \( (0, 36.8) \).

(b) Let \( t = 0 \): \( y = 1.64(0) + 36.8 = 36.8 \) 

(c) \( 65 = 1.64t \)

\[ t = \frac{65}{1.64} \approx 39.7 \]

This corresponds with the year 2017. 
Explanations will vary.

98. (a) The number reached 33 million in 1995.

(b) Part (a) can be solved graphically by graphing \( y = 0.39t + 31.0 \) and \( y = 33 \) on the same axes and finding the intersection, \( t = 5 \implies 1995 \). Part (a) can be solved algebraically by solving \( 0.39t + 31.0 = 33 \) for \( t \).

99. \( 10,000 = 0.32m + 2500 \)

\[ 7500 = 0.32m \]

\[ \frac{7500}{0.32} = m \]

\[ m = 23,437.5 \text{ miles} \]

100. \( y = -0.25t + 8 \)

\[ 1 = -0.25t + 8 \]

\[ 0.25t = 7 \]

\[ t = 28 \text{ hours} \]

101. False. \( x(3 - x) = 10 \implies 3x - x^2 = 10 \)

This is a quadratic equation. The equation cannot be written in the form \( ax + b = 0 \).

102. False. If both sides of the equation are graphed, you can see that they intersect, which means the equation has a real solution.

103. Equivalent equations are derived from the substitution principle and simplification techniques. They have the same solution(s).

\( 2x + 3 = 8 \) and \( 2x = 5 \) are equivalent equations.

104. To transform an equation into an equivalent equation, you should first remove symbols of grouping, combine like terms, and reduce fractions. Then, as needed, you may add (or subtract) the same quantity to (from) both sides of the equation, multiply (divide) both sides of the equation by the same nonzero quantity, or interchange the two sides of the equation.

105. (a)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -1 )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3.2x - 5.8 )</td>
<td>-9</td>
<td>-5.8</td>
<td>-2.6</td>
<td>0.6</td>
<td>3.8</td>
<td>7</td>
</tr>
</tbody>
</table>

(b) Since the sign changes from negative at 1 to positive at 2, the root is somewhere between 1 and 2.

\( 1 < x < 2 \)

(c)

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3.2x - 5.8 )</td>
<td>-1</td>
<td>-0.68</td>
<td>-0.36</td>
<td>-0.04</td>
<td>0.28</td>
<td>0.6</td>
</tr>
</tbody>
</table>

(d) Since the sign changes from negative at 1.8 to positive at 1.9, the root is somewhere between 1.8 and 1.9.

\( 1.8 < x < 1.9 \)

To improve accuracy, evaluate the expression at subintervals within this interval and determine where the sign changes.
106. \(0.3(x - 1.5) - 2 = 0\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3(x - 1.5) - 2</td>
<td>-0.65</td>
<td>-0.35</td>
<td>-0.05</td>
<td>0.25</td>
<td>0.55</td>
</tr>
</tbody>
</table>

The solution to \(0.3(x - 1.5) - 2 = 0\) is in the interval \(8 < x < 9\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>8.1</th>
<th>8.2</th>
<th>8.3</th>
<th>8.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3(x - 1.5) - 2</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>0.07</td>
</tr>
</tbody>
</table>

The solution is in the interval \(8.1 < x < 8.2\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>8.14</th>
<th>8.15</th>
<th>8.16</th>
<th>8.17</th>
<th>8.18</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3(x - 1.5) - 2</td>
<td>-0.008</td>
<td>-0.005</td>
<td>-0.002</td>
<td>0.001</td>
<td>0.004</td>
</tr>
</tbody>
</table>

The solution is in the interval \(8.16 < x < 8.17\).

107. \(\frac{x^2 + 5x - 36}{2x^2 + 17x - 9} = \frac{(x + 9)(x - 4)}{(2x - 1)(x + 9)} = \frac{x - 4}{2x - 1}, \ x \neq -9\)

108. \(\frac{x^2 - 49}{x^3 + x^2 + 3x - 21}\)

cannot be simplified.

109. \(y = 3x - 5\)

Intercepts: \((0, -5), \left(\frac{5}{3}, 0\right)\)

110. \(y = -\frac{1}{2}x - \frac{9}{2}\)

Intercepts: \((0, -\frac{9}{2}), (-9, 0)\)

111. \(y = -x^2 - 5x = -x(x + 5)\)

Intercepts: \((0, 0), (-5, 0)\)

112. \(y = \sqrt{5 - x}\)

Intercepts: \((5, 0), (0, \sqrt{5})\)
Section 1.3  Modeling with Linear Equations

- You should be able to set up mathematical models to solve problems.
- You should be able to translate key words and phrases.

(a) Equality:
- Equals, equal to, is, are, was, will be, represents

(b) Addition:
- Sum, plus, greater, increased by, more than, exceeds, total of

(c) Subtraction:
- Difference, minus, less than, decreased by, subtracted from, reduced by, the remainder

(d) Multiplication:
- Product, multiplied by, twice, times, percent of

(e) Division:
- Quotient, divided by, ratio, per

(f) Consecutive:
- Next, subsequent

- You should know the following formulas:

(a) Perimeter:
1. Square: \( P = 4s \)
2. Rectangle: \( P = 2l + 2w \)
3. Circle: \( C = 2\pi r \)
4. Triangle: \( P = a + b + c \)

(b) Area:
1. Square: \( A = s^2 \)
2. Rectangle: \( A = lw \)
3. Circle: \( A = \pi r^2 \)
4. Triangle: \( A = \frac{1}{2}bh \)

(c) Volume
1. Cube: \( V = s^3 \)
2. Rectangular solid: \( V = lwh \)
3. Cylinder: \( V = \pi r^2h \)
4. Sphere: \( V = \frac{4}{3}\pi r^3 \)

(d) Simple Interest: \( I = Prt \)

(e) Compound Interest: \( A = \left(1 + \frac{r}{n}\right)^{nt} \)

(f) Distance: \( d = rt \)

(g) Temperature: \( F = \frac{9}{5}C + 32 \)

- You should be able to solve word problems. Study the examples in the text carefully.

Vocabulary Check

1. mathematical modeling
2. verbal model; algebraic equation
3. \( A = \pi r^2 \)
4. \( P = 2l + 2w \)
5. \( V = s^3 \)
6. \( V = \pi r^2h \)
7. \( A = \left(1 + \frac{r}{12}\right)^{12t} \)
8. \( I = Prt \)

1. \( x + 4 \)
   - The sum of a number and 4
   - A number increased by 4

2. \( t - 10 \)
   - A number decreased by 10
   - The difference of a number and 10

3. \( \frac{u}{5} \)
   - The ratio of a number and 5
   - The quotient of a number and 5
   - A number divided by 5
4. \( \frac{2}{3}x \) 
The product of \( \frac{2}{3} \) and a number 
Two-thirds of a number

5. \( \frac{y - 4}{5} \) 
The difference of a number and 4 is divided by 5. 
A number decreased by 4 is divided by 5.

6. \( \frac{z + 10}{7} \) 
The sum of a number and 10 is divided by 7. 
A number increased by 10 is divided by 7.

7. \(-3(b + 2)\) 
The product of \(-3\) and the sum of a number and 2 
Negative 3 is multiplied by a number increased by 2.

8. \(\frac{-5(x - 1)}{8}\) 
The product of \(-5\) and the difference of a number and 1 is divided by 8. 
Negative 5 is multiplied by a number decreased by 1 and is divided by 8.

9. \(12x(x - 5)\) 
The difference of a number and 5 is multiplied by 12 times the number. 
12 is multiplied by a number and that product is multiplied by the number decreased by 5.

10. \(\frac{(q + 4)(3 - q)}{2q}\) 
The product of the sum of a number and 4 and the difference of 3 and a number is divided by the product of 2 and a number. 
A number increased by 4 is multiplied by 3 decreased by a number and the product is divided by 2 multiplied by a number.

11. Verbal Model: (Sum) = (first number) + (second number) 
Labels: Sum = \(S\), first number = \(n\), second number = \(n + 1\) 
Expression: \(S = n + (n + 1) = 2n + 1\)

12. Verbal Model: Product = (first number) \(\cdot\) (second number) 
Labels: Product = \(P\), first number = \(n\), second number = \(n + 1\) 
Expression: \(P = n(n + 1) = n^2 + n\)

13. Verbal Model: Product = (first odd integer)(second odd integer) 
Labels: Product = \(P\), first odd integer = \(2n - 1\), second odd integer = \(2n - 1 + 2 = 2n + 1\) 
Expression: \(P = (2n - 1)(2n + 1) = 4n^2 - 1\)

14. Verbal Model: (Sum) = (first even number)\(^2\) + (second even number)\(^2\) 
Labels: Sum = \(S\), first even number = \(2n\), second even number = \(2n + 2\) 
Expression: \(S = (2n)^2 + (2n + 2)^2 = 4n^2 + 4n^2 + 8n + 4 = 8n^2 + 8n + 4\)

15. Verbal Model: (Distance) = (rate) \(\cdot\) (time) 
Labels: Distance = \(d\), rate = 50 mph, time = \(t\) 
Expression: \(d = 50t\)

16. Verbal Model: (Distance) = (rate) \(\cdot\) (time) 
Labels: Distance = 200 km, rate = \(r\), time = \(t\) 
Expression: \(200 = rt, \ t = 200/r\)

17. Verbal Model: (Amount of acid) = 20% \(\cdot\) (amount of solution) 
Labels: Amount of acid (in gallons) = \(A\), amount of solution (in gallons) = \(x\) 
Expression: \(A = 0.20x\)
18. **Verbal Model:** (Sale price) = (list price) − (discount)

*Labels:* Sale price = S, list price = L, discount = 0.2L

*Expression:* \( S = L - 0.2L = 0.8L \)

19. **Verbal Model:** Perimeter = 2(width) + 2(length)

*Labels:* Perimeter = P, width = x, length = 2(width) = 2x

*Expression:* \( P = 2x + 2(2x) = 6x \)

20. **Verbal Model:** (Area) = \( \frac{1}{2} \) (base)(height)

*Labels:* Area = A, base = 20 inches, height = h inches

*Expression:* \( A = \frac{1}{2}(20)h = 10h \)

21. **Verbal Model:** (Total cost) = (unit cost)(number of units) + (fixed cost)

*Labels:* Total cost = C, fixed cost = $1200, unit cost = $25, number of units = x

*Expression:* \( C = 25x + 1200 \)

22. **Verbal Model:** (Revenue) = (price)(number of units)

*Labels:* Revenue = R, price = 3.59, number of units = x

*Expression:* \( R = 3.59x \)

23. **Verbal Model:** Thirty percent of the list price L

*Expression:* \( 0.30L \)

24. **Verbal Model:** 35% of \( q \)

*Expression:* \( 0.35q \)

25. **Verbal Model:** percent of 500 that is represented by the number \( N \)

*Equation:* \( N = p(500), \) \( p \) is in decimal form

26. (\( S_2 - S_1 \)) as a percent of \( S_1 \)

\( \frac{S_2 - S_1}{S_1} \) (100)

27. \[
\begin{array}{c}
4 \\
\hline
4 \\
\hline
x \\
\hline
x \\
\hline
2x \\
\hline
8
\end{array}
\]

Area = Area of top rectangle + Area of bottom rectangle

\( A = 4x + 8x = 12x \)

28. Area = \( \frac{1}{2} \) (base)(height)

\( \text{Area} = \frac{1}{2}b\left(\frac{3b}{2} + 1\right) = \frac{3}{4}b^2 + \frac{b}{2} \)

29. **Verbal Model:** Sum = (first number) + (second number)

*Labels:* Sum = 525, first number = \( n \), second number = \( n + 1 \)

*Equations:* \( 525 = n + (n + 1) \)

\( 525 = 2n + 1 \)

\( 524 = 2n \)

\( n = 262 \)

*Answer:* First number = \( n = 262 \), second number = \( n + 1 = 263 \)
30. **Verbal Model:** (Sum) (first number) + (second number) + (third number)

**Labels:** Sum = 804, first number = n, second number = n + 1, third number = n + 2

**Equation:**
- 804 = n + n + 1 + n + 2
- 804 = 3n + 3
- 801 = 3n
- 267 = n

**Answer:** n = 267, n + 1 = 268 (second number), and n + 2 = 269 (third number)

31. **Verbal Model:** Difference = (one number) − (another number)

**Labels:** Difference = 148, one number = 5x, another number = x

**Equation:**
- 148 = 5x − x
- 148 = 4x
- x = 37
- 5x = 185

**Answer:** The two numbers are 37 and 185.

32. **Verbal Model:** (Difference) = (number) − (one-fifth of number)

**Labels:** Difference = 76, number = n, one-fifth of number = \( \frac{1}{5}n \)

**Equation:**
- 76 = n − \( \frac{1}{5}n \)
- 76 = \( \frac{4}{5}n \)
- 95 = n

**Answer:** The numbers are 95 and \( \frac{1}{5} \cdot 95 = 19 \).

33. **Verbal Model:** Product = (smaller number) · (larger number) = (smaller number)\(^2\) − 5

**Labels:** Smaller number = n, larger number = n + 1

**Equation:**
- n(n + 1) = \( n^2 \) − 5
- \( n^2 + n = n^2 \) − 5
- n = −5

**Answer:** Smaller number = n = −5, larger number = n + 1 = −4

34. **Verbal Model:** Difference = (reciprocal of smaller number) − (reciprocal of larger number) = \( \frac{1}{4} \) (reciprocal of smaller number)

**Labels:** Smaller number = n, larger number = n + 1, difference = \( \frac{1}{4n} \)

**Equation:**
- \( \frac{1}{4n} = \frac{1}{n} - \frac{1}{n + 1} \)

Multiply both sides by 4n(n + 1).

\[ 4n(n + 1) \cdot \frac{1}{4n} = 4n(n + 1) \cdot \frac{1}{n} - 4n(n + 1) \cdot \frac{1}{n + 1} \]

\[ n + 1 = 4(n + 1) - 4n \]

\[ n + 1 = 4n + 4 - 4n \]

\[ n = 3 \]

**Answer:** The numbers are 3 and n + 1 = 4.
35. \[ x = \text{percent} \cdot \text{number} \]
\[ = (30\%)(45) \]
\[ = 0.30(45) \]
\[ = 13.5 \]

36. \[ x = \text{percent} \cdot \text{number} \]
\[ = (175\%)(360) \]
\[ = (1.75)(360) \]
\[ = 630 \]

37. \[ 432 = \text{percent} \cdot 1600 \]
\[ 432 = p(1600) \]
\[ \frac{432}{1600} = p \]
\[ p = 0.27 = 27\% \]

38. \[ 459 = \text{percent} \cdot 340 \]
\[ 459 = p(340) \]
\[ \frac{459}{340} = p \]
\[ p = 1.35 = 135\% \]

39. \[ 12 = \frac{1}{2} \% \cdot \text{number} \]
\[ 12 = 0.005x \]
\[ \frac{12}{0.005} = x \]
\[ x = 2400 \]

40. \[ 70 = 40\% \cdot \text{number} \]
\[ \text{number} = \frac{70}{40\%} = \frac{70}{0.40} = 175 \]

41. **Verbal Model:** Loan payments = 58.6\% \cdot \text{Annual income}

**Labels:**
- Loan payments = \( I \)
- Annual income = \( I \)

**Equation:**
\[ 13.077.75 = 0.586 I \]
\[ I \approx 22,316.98 \]

The family’s annual income is $22,316.98.

42. (a) Total income = \( 148.0 + 858.3 + 700.8 + 146.1 = $1853.2 \) billion

- Corporate taxes: \( \frac{148.0}{1853.2} \approx 0.080 \) or 8\%
- Income tax: \( \frac{858.3}{1853.2} \approx 0.463 \) or 46.3\%
- Social Security: \( \frac{700.8}{1853.2} \approx 0.378 \) or 37.8\%
- Other: \( \frac{146.1}{1853.2} \approx 0.079 \) or 7.9\%

(b) Total expenses = \( 171.0 + 1317.9 + 348.6 + 173.5 = $2011 \) billion

- Interest on debt: \( \frac{171}{2011} = 0.085 = 8.5\% \)
- Health and human services: \( \frac{1317.9}{2011} = 0.65535 = 65.5\% \)
- Defense department: \( \frac{348.6}{2011} = 0.1734 = 17.3\% \)
- Other: \( \frac{173.5}{2011} = 0.0863 = 8.6\% \)

(c) There is a deficit of \( 1853.2 - 2011.0 = -$157.8 \) billion.
43. **Verbal Model:** (2002 price for a gallon of unleaded gasoline) = (percentage increase)(1990 price for a gallon of unleaded gasoline) + (1990 price for a gallon of unleaded gasoline)

**Labels:**
- 2002 price for a gallon of unleaded gasoline: $1.36
- 1990 price for a gallon of unleaded gasoline: $1.16
- Percentage increase: \( p \)

**Equation:**
- \( 1.36 = 1.16p + 1.16 \)
- \( 0.20 = 1.16p \)
- \( 0.172 = p \)

**Answer:** Percentage increase \( \approx 17.2\% \)

44. **Verbal Model:** (2002 price) = (percent increase)(1990 price) + 1990 price

**Labels:**
- 2002 price = $3.76, percent increase = \( p \), 1990 price = $2.54

**Equation:**
- \( 3.76 = 2.54p + 2.54 \)
- \( 1.22 = 2.54p \)
- \( p = \frac{1.22}{2.54} = 0.48 \)

The percent increase in the price of a half-gallon of ice cream from 1990 to 2002 is 48%.

45. **Verbal Model:** (2002 price for a pound of tomatoes) = (percentage increase)(1990 price for a pound of tomatoes) + (1990 price for a pound of tomatoes)

**Labels:**
- 2002 price for a pound of tomatoes: $1.66
- 1990 price for a pound of tomatoes: $0.86
- Percentage increase: \( p \)

**Equation:**
- \( 1.66 = 0.86p + 0.86 \)
- \( 0.80 = 0.86p \)
- \( 0.930 = p \)

**Answer:** Percentage increase \( \approx 93\% \)

46. **Verbal Model:** (2002 price) = (percent increase)(1990 price) + 1990 price

**Labels:**
- 2002 price = $855, percent increase = \( p \), 1990 price = $1050

**Equation:**
- \( 855 = 1050p + 1050 \)
- \( 195 = 1050p \)
- \( p = \frac{195}{1050} = -0.1857 \approx 18.6\% \) decrease

The percent decrease in the price of a personal computer from 1990 to 2002 is 18.6%.

47. **Verbal Model:** (Sale price) = (list price) − (discount)

**Labels:**
- Sale price = $1210.75, list price = \( L \), discount = 0.165\( L \)

**Equation:**
- \( 1210.75 = L - 0.165L \)
- \( 1210.75 = 0.835L \)
- \( 1450 = L \)

**Answer:** The list price of the pool is $1450.
48. Verbal Model: \((\text{Total}) = (\text{salesperson’s paycheck}) + (\text{coworker’s paycheck})\)

Labels: Total = $645, coworker’s paycheck = \(x\), salesperson’s paycheck = \(x - 0.15x = 0.85x\)

Equation: 
\[
645 = 0.85x + x \\
645 = 1.85x \\
348.65 \approx x
\]

Coworker’s paycheck is $348.65 and salesperson’s paycheck is \(0.85x = 296.35\).

49. (a) 
\[
\begin{array}{c}
| \ \\
| \ \\
| \ \\
| \ \\
\end{array}
\]

(b) \(l = 1.5w\)

\[
P = 2l + 2w \\
= 2(1.5w) + 2w \\
= 5w
\]

Width: \(w = 5\) meters

Length: \(l = 1.5w = 7.5\) meters

Dimensions: \(7.5\) meters \(\times\) \(5\) meters

50. (a) 
\[
\begin{array}{c}
| \ \\
| \ \\
| \ \\
| \ \\
\end{array}
\]

(b) \(h = 0.62w\)

\[
P = 2h + 2w \\
= 2(0.62w) + 2w \\
= 3.24w
\]

Width: \(w = 0.62\) meters

Height: \(h = 0.62w = 0.38\) meters

Dimensions: \(0.38\) m \(\times\) \(0.62\) m

51. Verbal Model: \(\text{Average} = \frac{(\text{test #1}) + (\text{test #2}) + (\text{test #3}) + (\text{test #4})}{4}\)

Labels: Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = \(x\)

Equation: 
\[
90 = \frac{87 + 92 + 84 + x}{4} \\
360 = 263 + x \\
x = 97
\]

Answer: You must score 97 (or better) on test #4 to earn an A for the course.

52. Verbal Model: \(\text{Average} = \frac{(\text{test #1}) + (\text{test #2}) + (\text{test #3}) + (\text{test #4})}{5}\)

Labels: Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = \(x\)

Equation: 
\[
90 = \frac{87 + 92 + 84 + x}{5} \\
450 = 87 + 92 + 84 + x \\
450 = 263 + x \\
x = 187
\]

You must score 187 out of 200 on the last test to get an A in the course.

53. Rate = \(\frac{\text{distance}}{\text{time}}\) = \(\frac{50}{\frac{1}{2}}\) = 100 kilometers/hour

Total time = \(\frac{\text{total distance}}{\text{rate}}\) = \(\frac{300}{100}\) = 3 hours
54. Distance = rate \cdot time
\[ d_1 = 40 \text{ mph} \cdot t \]
\[ d_2 = 55 \text{ mph} \cdot t \]

(Distance between cars) = (second distance) − (first distance)
\[ S = d_2 - d_1 \]
\[ S = 55t - 40t = 15t \]
\[ t = \frac{1}{3} \text{ hour} \]

\( \frac{1}{3} \text{ hour} \) (or 20 minutes) must elapse before the two cars are 5 miles apart.

55. (time on first part) + (time on second part) = (Total time)
\[ t_1 + t_2 = T \]
\[ \frac{d_1}{r_1} + \frac{d_2}{r_2} = T \]
\[ \frac{x}{58} + \frac{317 - x}{52} = 5.75 \]
Multiply both sides by 58(52).
\[ 52x + 58(317 - x) = 5.75(58)(52) \]
\[ 52x + 18,386 - 58x = 17,342 \]
\[ -6x = -1044 \]
\[ x = 174 \text{ miles} \]
\[ t_1 = \frac{174}{58} = 3 \text{ hours} \]
\[ t_2 = \frac{317 - 174}{52} = 2.75 \text{ hours} \]

The salesman averaged 58 miles per hour for 3 hours and 52 miles per hour for 2 hours and 45 minutes.

56. **Verbal Model:** (Distance traveled by first car) = (distance traveled by second car)

**Labels:** Distance traveled by first car = 45t, distance traveled by second car = 55(t - \( \frac{1}{3} \))

**Equation:**
\[ 45t = 55(t - \frac{1}{3}) \]
\[ 45t = 55t - 27.5 \]
\[ -10t = -27.5 \]
\[ t = 2.75 \]

The second car catches up to the first car after 2.75 hours (or 2 hours and 45 minutes). The distance traveled is 45(2.75) = 123.75 miles. Thus, the second car catches up before the first car arrives at the game.

57. (a) Time for the first family: \( t_1 = \frac{d}{r_1} = \frac{160}{42} \approx 3.81 \text{ hours} \)

Time for the other family: \( t_2 = \frac{d}{r_2} = \frac{160}{50} = 3.2 \text{ hours} \)

(b) \( t = \frac{d}{r} = \frac{100}{42 + 50} = \frac{100}{92} \approx 1.08 \text{ hours} \)

(c) \( d = rt = 42(\frac{160}{42} - \frac{160}{50}) = 25.6 \text{ miles} \)
58. **Verbal Model:** (Distance) = (rate)(time₁ + time₂)

**Labels:**
- Distance = 2 \cdot 200 = 400 miles, rate = r,
- time₁ = \frac{distance}{rate₁} = \frac{200}{55} hours,
- time₂ = \frac{distance}{rate₂} = \frac{200}{40} hours

**Equation:**
\[
400 = r \left(\frac{200}{55} + \frac{200}{40}\right) \\
400 = r \left(\frac{1600}{440} + \frac{2200}{440}\right) = \frac{3800}{440} \cdot r \\
46.3 \approx r
\]

The average speed for the round trip was approximately 46.3 miles per hour.

59. **Verbal Model:** time = \frac{\text{distance}}{\text{rate}}

**Labels:**
- Let x = wind speed, then the rate to the city = 600 + x, the rate from the city = 600 - x, the distance to the city = 1500 kilometers, the distance traveled so far in the return trip = 1500 - 300 = 1200 kilometers.

**Equation:**
\[
\begin{align*}
1500 & = 600 + x & & \text{Wind speed: } 66\frac{2}{3} \text{ kilometers per hour} \\
900,000 - 1500x & = 720,000 + 1200x \\
180,000 & = 2700x \\
\frac{2}{3} & = x
\end{align*}
\]

60. **Verbal Model:** (Distance) = (rate)(time)

**Labels:**
- Distance = 1.5 \times 10^{11} meters
- Rate = 3.0 \times 10^8 meters per second
- Time = t

**Equation:**
\[
1.5 \times 10^{11} = (3.0 \times 10^8) t \\
500 = t
\]

Light from the sun travels to the earth in 500 seconds or approximately 8.33 minutes.

61. **Verbal Model:** time = \frac{\text{distance}}{\text{rate}}

**Equation:**
\[
t = \frac{3.84 \times 10^8 \text{ meters}}{3.0 \times 10^8 \text{ meters per second}} \\
t = 1.28 \text{ seconds}
\]

The radio wave travels from Mission Control to the moon in 1.28 seconds.

62. **Verbal Model:** \frac{\text{height of tree}}{\text{height of tree’s shadow}} = \frac{\text{height of lamppost}}{\text{height of lamppost’s shadow}}

**Labels:**
- Height of tree = h, height of tree’s shadow = 8 meters,
- height of lamppost = 2 meters, height of lamppost’s shadow = 0.75 meter

**Equation:**
\[
\frac{h}{8} = \frac{2}{0.75} \\
h = \frac{8(2)}{0.75} = 21\frac{1}{2}
\]

The tree is 21\frac{1}{2} meters tall.

63. **Verbal Model:** \frac{\text{height of building}}{\text{length of building’s shadow}} = \frac{\text{height of stake}}{\text{length of stake’s shadow}}

**Label:**
Let h = height of the building in feet.

**Equation:**
\[
\frac{h}{87} = \frac{4}{1/3} \\
\frac{1}{3}h = 348 \\
h = 1044 \text{ feet}
\]

The Chrysler building is 1044 feet tall.
64. (a) \[ h = \frac{30}{5} + 5 = 35 \text{ feet} \]

(b) **Verbal Model:**

\[
\frac{\text{height of pole}}{\text{height of pole's shadow}} = \frac{\text{height of person}}{\text{height of person's shadow}}
\]

**Labels:**

Height of pole = \( h \), height of pole’s shadow = 30 + 5 = 35 feet, height of person = 6 feet, height of person’s shadow = 5 feet

**Equation:**

\[
\frac{h}{35} = \frac{6}{5}
\]

\[
h = \frac{6}{5} \times 35 = 42
\]

The pole is 42 feet tall.

65. **Verbal Model:**

\[
\frac{\text{height of silo}}{\text{length of silo's shadow}} = \frac{\text{height of person}}{\text{height of person's shadow}}
\]

**Label:**

Let \( x \) = length of person’s shadow.

**Equation:**

\[
\frac{50}{32 + x} = \frac{6}{x}
\]

\[
50x = 6(32 + x)
\]

\[
50x = 192 + 6x
\]

\[
44x = 192
\]

\[
x = 4.36 \text{ feet}
\]

66. **Verbal Model:**

\[
(\text{Interest from 4\frac{1}{2}\%}) + (\text{interest from 5\%}) = (\text{total interest})
\]

**Labels:**

Amount invested at 4\frac{1}{2}\% = \( x \), amount invested at 5\% = 12,000 – \( x \), interest from 4\frac{1}{2}\% = \( x \)(0.045), interest from 5\% = (12,000 – \( x \))(0.05), total annual interest = $580

**Equation:**

\[
0.045x + 0.05(12,000 - x) = 580
\]

\[
0.045x + 600 - 0.05x = 580
\]

\[-0.005x = -20
\]

\[
x = 4000
\]

The smallest amount that can be invested at 5\% is 12,000 – 4000 = $8000.

67. **Verbal Model:**

\[(\text{Interest in 3\% fund}) + (\text{interest in 4\frac{1}{2}\% fund}) = (\text{total interest})\]

**Labels:**

Let \( x \) = amount in the 3\% fund. Then 25,000 – \( x \) = amount in the 4\frac{1}{2}\% fund.

**Equation:**

\[
1000 = 0.03x + 0.045(25,000 - x)
\]

\[
1000 = 0.03x + 1125 - 0.045x
\]

\[-125 = -0.015x
\]

\[
x = 8333.33 \text{ at 3\%}
\]

\[
25,000 - x = 16,666.67 \text{ at 4\frac{1}{2}\%}
\]

68. **Verbal Model:**

\[(\text{Profit from dogwood trees}) + (\text{profit from red maple trees}) = (\text{total profit})\]

**Labels:**

Inventory of dogwood trees = \( x \), inventory of red maple trees = 20,000 – \( x \),
profit from dogwood trees = 0.25\( x \), profit from red maple trees = 0.17(20,000 – \( x \)),
total profit = 0.20(20,000) = 4000

**Equation:**

\[
0.25x + 0.17(20,000 - x) = 4000
\]

\[
0.25x + 3400 - 0.17x = 4000
\]

\[
0.08x = 600
\]

\[
x = 7500
\]

The amount invested in dogwood trees was $7500 and the amount invested in red maple trees was 20,000 – 7500 = $12,500.
69. **Verbal Model:** (profit on minivans) + (profit on SUVs) = (total profit)

**Labels:** Let \( x \) = amount invested in minivans. Then, \( 600,000 - x \) = amount invested in SUVs.

**Equation:**

\[
0.24x + 0.28(600,000 - x) = 0.25(600,000)
0.24x + 168,000 - 0.28x = 150,000
-0.04x = -18,000
x = 450,000
\]

$450,000 is invested in minivans and \( 600,000 - x \) = $150,000 is invested in SUVs.

70. (a) **Verbal Model:** \( \left( \frac{\text{Amount in}}{\text{Solution 1}} \right) + \left( \frac{\text{amount in}}{\text{Solution 2}} \right) = \left( \frac{\text{amount in}}{\text{final solution}} \right) \)

**Labels:** Amount in Solution 1 = 0.10x, amount in Solution 2 = 0.30(100 - x), amount in final solution = 0.25(100)

**Equation:**

\[
0.10x + 0.30(100 - x) = 0.25(100)
0.10x + 30 - 0.30x = 25
-0.20x = -5
x = 25
\]

Use 25 gallons of Solution 1 and 100 - 25 = 75 gallons of Solution 2.

(b) **Verbal Model:** \( \left( \frac{\text{Amount in}}{\text{Solution 1}} \right) + \left( \frac{\text{amount in}}{\text{Solution 2}} \right) = \left( \frac{\text{amount in}}{\text{final solution}} \right) \)

**Labels:** Amount in Solution 1 = 0.25x, amount in Solution 2 = 0.50(5 - x), amount in final solution = 0.30(5)

**Equation:**

\[
0.25x + 0.50(5 - x) = 1.5
0.25x + 2.5 - 0.50x = 1.5
-0.25x = -1
x = 4
\]

Use 4 liters of Solution 1 and 5 - 4 = 1 liter of Solution 2.

(c) **Verbal Model:** \( \left( \frac{\text{Amount in}}{\text{Solution 1}} \right) + \left( \frac{\text{amount in}}{\text{Solution 2}} \right) = \left( \frac{\text{amount in}}{\text{final solution}} \right) \)

**Labels:** Amount in Solution 1 = 0.15x, amount in Solution 2 = 0.45(10 - x), amount in final solution = 0.30(10)

**Equation:**

\[
0.15x + 0.45(10 - x) = 0.30(10)
0.15x + 4.5 - 0.45x = 3
-0.30x = -1.5
x = 5
\]

Use 5 quarts of Solution 1 and 10 - 5 = 5 quarts of Solution 2.

(d) **Verbal Model:** \( \left( \frac{\text{Amount in}}{\text{Solution 1}} \right) + \left( \frac{\text{amount in}}{\text{Solution 2}} \right) = \left( \frac{\text{amount in}}{\text{final solution}} \right) \)

**Labels:** Amount in Solution 1 = 0.70x, amount in Solution 2 = 0.90(25 - x), amount in final solution = 0.75(25)

**Equation:**

\[
0.70x + 0.90(25 - x) = 0.75(25)
0.70x + 22.5 - 0.90x = 18.75
-0.20x = -3.75
x = 18.75
\]

Use 18.75 gallons of Solution 1 and 25 - 18.75 = 6.25 gallons of Solution 2.
71. **Verbal Model:** \((\text{Final concentration})(\text{Amount}) = (\text{Solution 1 concentration})(\text{Amount}) + (\text{Solution 2 concentration})(\text{Amount})\)

**Label:** Let \(x\) = amount of 100% concentrate

**Equation:**
\[
0.75(55) = 0.40(55 - x) + 1.00x \\
41.25 = 22 - 0.40x + 1.00x \\
41.25 = 0.60x + 22 \\
x = \frac{32.1\text{ gallons}}{110}
\]

Approximately 32.1 gallons of the 100% concentrate will be needed.

72. **Verbal Model:** \(\left(\frac{\text{Amount of gasoline}}{\text{in Solution 1}}\right) + \left(\frac{\text{Amount of gasoline}}{\text{in Solution 2}}\right) = \left(\frac{\text{Amount of gasoline}}{\text{in final solution}}\right)\)

**Labels:** Amount of gasoline in Solution 1 = \(\frac{64}{33}\), amount of gasoline in Solution 2 = \(x\), amount of gasoline in final solution = \(\frac{1353}{41}\)

**Equation:**
\[
\frac{64}{33} + x = \frac{40}{41}(2 + x) \\
\frac{64}{33} + x = \frac{40}{33} + \frac{40}{41}x \\
\text{Multiply both sides by } 33(41).
\]
\[
2624 + 1353x = 2640 + 1320x \\
33x = 16 \\
x = \frac{0.48}{110}\text{ gallon of gasoline}
\]

Approximately 0.48 gallon of gasoline must be added.

73. **Verbal Model:** \((\text{price per pound of peanuts})(\text{number of pounds of peanuts}) + (\text{price per pound of walnuts})(\text{number of pounds of walnuts}) + (\text{price per pound of nut mixture})(\text{number of pounds of nut mixture}) = (\text{price per pound of nut mixture})(\text{number of pounds of nut mixture})\)

**Labels:** Let \(x\) = number of pounds of $2.49 peanuts. Then \(100 - x\) = number of pounds of $3.89 walnuts.

**Equation:**
\[
2.49x + 3.89(100 - x) = 3.19(100) \\
2.49x + 389 - 3.89x = 319 \\
-1.40x = -70 \\
x = \frac{-70}{-1.40} \\
x = 50\text{ pounds of }$2.49\text{ peanuts} \\
100 - x = 50\text{ pounds of }$3.89\text{ walnuts}
\]

Use 50 pounds of peanuts and 50 pounds of walnuts.

74. **Verbal Model:** \((\text{Fixed costs}) + (\text{variable cost per unit})(\text{number of units}) = (\text{total costs})\)

**Labels:** Fixed costs = $10,000, variable costs = $8.50, total costs = $85,000

**Equation:**
\[
10,000 + 8.50x = 85,000 \\
8.50x = 75,000 \\
x = \frac{75,000}{8.50} = 8823.53
\]

The company can produce no more than 8823 units with $85,000 budgeted monthly for costs.

75. **Verbal Model:** Total cost = \((\text{Fixed cost}) + (\text{Variable cost per unit}) \times (\text{Number of units})\)

**Labels:** Total cost = $85,000, variable costs = $9.30, fixed costs = $10,000

**Equation:**
\[
85,000 = 10,000 + 9.30x \\
x = \frac{75,000}{9.3} = 8064.52\text{ units}
\]

At most the company can manufacture 8064 units.
76. Verbal Model: (Volume) = (length)(width)(height)

Labels:  
Volume = 70 gallons \times 0.13368 cubic feet  
= 9.3576 cubic feet,  
length = 12 feet, width = 3 feet,  
height = \( h \)

Equation:  
9.3576 = (12)(3)h

9.3576 = 36h

0.26 \approx h

The depth of the water is 0.26 foot.

77. \( A = \frac{1}{2}bh \)

78. \( A = \frac{1}{2}(a + b)h \)

\[ 2A = (a + b)h \]

\[ \frac{2A}{h} = a + b \]

\[ b = \frac{2A}{h} - a \]

79. \( S = C + RC \)

\[ S = C(1 + R) \]

\[ \frac{S}{1 + R} = C \]

80. \( A = P + Prt \)

\[ A - P = Prt \]

\[ \frac{A - P}{Prt} = r \]

81. \( V = \frac{4}{3} \pi a^2 b \)

\[ \frac{3}{4}V = \pi a^2 b \]

\[ \frac{(3/4)V}{\pi a^2} = b \]

\[ \frac{3V}{4\pi a^2} = b \]

82. \( V = \frac{1}{3} \pi h^2(3r - h) \)

\[ 3V = \pi h^2(3r - h) \]

\[ 3V = 3 \pi h^2 - \pi h^3 \]

\[ 3V + \pi h^3 = 3 \pi rh^2 \]

\[ \frac{3V + \pi h^3}{3 \pi h^2} = r \]

83. \( h = v_d + \frac{1}{2}at^2 \)

\[ h - v_d = \frac{1}{2}at^2 \]

\[ 2(h - v_d) = at^2 \]

\[ \frac{2(h - v_d)}{t^2} = a \]

84. \( f = (n - 1)\left( \frac{1}{R_1} - \frac{1}{R_2} \right) \)

\[ \frac{1}{(n - 1)f} = \frac{1}{R_1} - \frac{1}{R_2} \]

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ \frac{1}{(n - 1)f} + \frac{1}{R_2} = \frac{1}{R_1} \]

\[ \frac{R_2 + (n - 1)f}{(n - 1)f R_2} = \frac{1}{R_1} \]

\[ R_1 = \frac{(n - 1)f R_2}{R_2 + (n - 1)f} \]

\( R_i \) is the reciprocal of \( \frac{1}{R_i} \).

85. \( C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \)

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ \frac{C_2 - C}{CC_2} = \frac{1}{C_1} \]

\[ \frac{CC_2}{C_2 - C} = C_1 \]

86. \( S = \frac{n^2}{2}[2a + (n - 1)d] \)

\[ \frac{2S}{n} = 2a + (n - 1)d \]

\[ \frac{2S}{n} - (n - 1)d = 2a \]

\[ \frac{2S - n(n - 1)d}{n} = 2a \]

\[ \frac{2S - n(n - 1)d}{2n} = a \]
87. \[ L = a + (n - 1)d \]
\[ L - a = nd - d \]
\[ L - a + d = nd \]
\[ \frac{L - a + d}{d} = n \]

88. \[ S = \frac{rL - a}{r - 1} \]
\[ S(r - 1) = rL - a \]
\[ Sr - S = rL - a \]
\[ r(S - L) = S - a \]
\[ r = \frac{S - a}{S - L} \]

90. \[ W_1x = W_2(L - x) \]
\[ W_1 = 200 \text{ pounds} \]
\[ W_2 = 550 \text{ pounds} \]
\[ L = 5 \text{ feet} \]
\[ 200x = 550(5 - x) \]
\[ 200x = 2750 - 550x \]
\[ 750x = 2750 \]
\[ x = \frac{2}{3} \text{ feet from the person} \]

91. \[ V = \frac{4}{3}\pi r^3 \]
\[ 5.96 = \frac{4}{3}\pi r^3 \]
\[ \frac{17.88}{4\pi} = r^3 \]
\[ r = \sqrt[3]{\frac{4.47}{\pi}} = 1.12 \text{ inches} \]

92. \[ V = \pi r^2h \]
\[ h = \frac{V}{\pi r^2} = \frac{603.2}{\pi(2)^2} = 48 \text{ feet} \]

93. \[ C = \frac{5}{9}(F - 32) \]
When \( F = 64.4^\circ \),
\[ \frac{5}{9}(64.4 - 32) = 18^\circ \text{C}. \]

94. \[ C = \frac{5}{9}(F - 32) \]
When \( F = 39.1^\circ \),
\[ \frac{5}{9}(39.1 - 32) = 3.9^\circ \text{C}. \]

95. \[ F = \frac{9}{5}C + 32 \]
When \( C = 50^\circ \),
\[ \frac{9}{5}(50) + 32 = 122^\circ \text{F}. \]

96. \[ F = \frac{9}{5}C + 32 \]
When \( C = -30^\circ \),
\[ \frac{9}{5}(-30) + 32 = -22^\circ \text{F}. \]

97. False, it should be written as
\[ \frac{z^3 - 8}{z^2 - 9} \]

98. False
Cube: \[ V = s^3 \]
\[ = 9.5^3 \]
\[ = 857.375 \text{ in.}^3 \]
Sphere: \[ V = \frac{4}{3}\pi r^3 \]
\[ = \frac{4}{3}\pi(5.9)^3 \]
\[ \approx 860.290 \text{ in.}^3 \]

99. \[ ax + b = 0 \implies x = -\frac{b}{a} \]
(a) If \( ab > 0 \), then \( a \) and \( b \) have the same sign and \( x = -b/a \) is negative.
(b) If \( ab < 0 \), then \( a \) and \( b \) have opposite signs and \( x = -b/a \) is positive.

100. Answers will vary. Sample answer: \( x + 7 = 4 \)

101. \( (5x^3)(25x^2)^{-1} = \frac{5x^5}{25x^2} = \frac{x^3}{5}, \ x \neq 0 \)

102. \( \sqrt{150x^7} = \sqrt{25 \cdot 6x^7} = 5st\sqrt{6t} \)
103. \[ \frac{3}{x - 5} + \frac{2}{5 - x} = \frac{3}{x - 5} + \frac{(-1)(2)}{x - 5} = \frac{3 - 2}{x - 5} = \frac{1}{x - 5} \]

104. \[ \frac{5}{x} + \frac{3x}{x^2 - 9} - \frac{10}{x + 3} = \frac{5(x + 3)(x - 3) + 3x^2 - 10(x - 3)}{x(x + 3)(x - 3)} \]
\[ = \frac{5(x^2 - 9) + 3x^2 - 10x + 30x}{x(x + 3)(x - 3)} \]
\[ = \frac{5x^2 - 45 + 3x^2 - 10x + 30x}{x(x + 3)(x - 3)} \]
\[ = \frac{-2x^2 + 30x - 45}{x(x + 3)(x - 3)} \]

105. \[ \frac{10}{\sqrt{3}} = \frac{10 \cdot \sqrt{3}}{\sqrt{3}} = \frac{10 \sqrt{3}}{21} \]

106. \[ \frac{4}{\sqrt{6}} = \frac{4 \cdot \sqrt{6}}{\sqrt{6}} = \frac{4 \sqrt{6}}{\sqrt{2 \cdot 16}} = \frac{4 \sqrt{6}}{6} = \frac{2 \sqrt{6}}{3} \]

107. \[ \frac{5}{\sqrt{6} + \sqrt{11}} = \frac{5}{\sqrt{6} + \sqrt{11}} \cdot \frac{\sqrt{6} - \sqrt{11}}{\sqrt{6} - \sqrt{11}} \]
\[ = \frac{5\sqrt{6} - 5\sqrt{11}}{-5} \]
\[ = -(\sqrt{6} - \sqrt{11}) \]
\[ = \sqrt{11} - \sqrt{6} \]

108. \[ \frac{14}{\sqrt{10} - 1} = \frac{14 \cdot \sqrt{10} + 1}{\sqrt{10} - 1} \]
\[ = \frac{14(3 \cdot \sqrt{10} + 1)}{(3 \cdot \sqrt{10})^2 - (1)^2} \]
\[ = \frac{14(3 \cdot \sqrt{10} + 1)}{9(10) - 1} \]
\[ = \frac{14(3 \cdot \sqrt{10} + 1)}{89} \]
\[ = \frac{42 \cdot \sqrt{10} + 14}{89} \]

**Section 1.4 Quadratic Equations and Applications**

- You should be able to solve a quadratic equation by factoring, if possible.
- You should be able to solve a quadratic equation of the form \( u^2 = d \) by extracting square roots.
- You should be able to solve a quadratic equation by completing the square.
- You should know and be able to use the Quadratic Formula: For \( ax^2 + bx + c = 0, a \neq 0 \),
  \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} . \]
- You should be able to determine the types of solutions of a quadratic equation by checking the discriminant \( b^2 - 4ac \).
  (a) If \( b^2 - 4ac > 0 \), there are two distinct real solutions. The graph has two \( x \)-intercepts.
  (b) If \( b^2 - 4ac = 0 \), there is one repeated real solution. The graph has one \( x \)-intercept.
  (c) If \( b^2 - 4ac < 0 \), there is no real solution. The graph has no \( x \)-intercepts.
- You should be able to use your calculator to solve quadratic equations.
- You should be able to solve applications involving quadratic equations. Study the examples in the text carefully.
Vocabulary Check

1. quadratic equation
2. factoring, extracting square roots, completing the square, and the Quadratic Formula
3. discriminant
4. position equation; \(-16t^2 + v_f t + s_0\); velocity of the object; initial height of the object
5. Pythagorean Theorem

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<td>(x^2 = 16x)</td>
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<td>General form: (13 - 3(x^2 + 14x + 49) = 0)</td>
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<td>5.</td>
<td>(\frac{1}{2}(3x^2 - 10) = 18x)</td>
<td>General form: (3x^2 - 10 = 90x)</td>
</tr>
<tr>
<td>6.</td>
<td>(x(x + 2) = 5x^2 + 1)</td>
<td>General form: (-4x^2 + 2x - 1 = 0)</td>
</tr>
<tr>
<td>7.</td>
<td>(6x^2 + 3x = 0)</td>
<td>General form: (-3x^2 - 42x - 134 = 0)</td>
</tr>
<tr>
<td>8.</td>
<td>(9x^2 - 1 = 0)</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>(x^2 - 2x - 8 = 0)</td>
<td>General form: (x^2 - 4x + 2 = 0)</td>
</tr>
<tr>
<td>10.</td>
<td>(x^2 - 10x + 25 = 0)</td>
<td>General form: (x^2 - 2x + 1 = 0)</td>
</tr>
<tr>
<td>11.</td>
<td>(x^2 + 10x + 25 = 0)</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>(4x^2 + 12x + 9 = 0)</td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>(3 + 5x - 2x^2 = 0)</td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>(2x^2 = 19x + 33)</td>
<td>General form: (2x^2 - 19x - 33 = 0)</td>
</tr>
<tr>
<td>15.</td>
<td>(x^2 + 4x = 12)</td>
<td>General form: (x^2 + 6x - 2 = 0)</td>
</tr>
<tr>
<td>16.</td>
<td>(-x^2 + 8x = 12)</td>
<td>General form: (-x^2 + 8x - 12 = 0)</td>
</tr>
<tr>
<td>17.</td>
<td>(\frac{1}{2}x^2 + 8x + 20 = 0)</td>
<td></td>
</tr>
<tr>
<td>18.</td>
<td>(\frac{1}{5}x^2 - x - 16 = 0)</td>
<td>General form: (x^2 - 8x - 128 = 0)</td>
</tr>
</tbody>
</table>
19. \(x^2 + 2ax + a^2 = 0\)

\[(x + a)^2 = 0\]

\[x + a = 0\]

\[x = -a\]

The only solution to the equation is \(x = -a\).

20. \((x + a)^2 - b^2 = 0\)

\[[(x + a) + b][(x + a) - b] = 0\]

\[x + a + b = 0 \implies x = -a - b\]

\[x + a - b = 0 \implies x = -a + b\]

21. \(x^2 = 49\)

\[x = \pm 7\]

22. \(x^2 = 169\)

\[x = \pm \sqrt{169} = \pm 13\]

23. \(x^2 = 11\)

\[x = \pm \sqrt{11}\]

24. \(x^2 = 32\)

\[x = \pm \sqrt{32} = \pm 4\sqrt{2}\]

25. \(3x^2 = 81\)

\[x^2 = 27\]

\[x = \pm 3\sqrt{3}\]

26. \(9x^2 = 36\)

\[x^2 = 4\]

\[x = \pm \sqrt{4} = \pm 2\]

27. \((x - 12)^2 = 16\)

\[x - 12 = \pm 4\]

\[x = 16 \text{ or } x = 8\]

28. \((x + 13)^2 = 25\)

\[x + 13 = \pm 5\]

\[x = -13 \pm 5 = -8, -18\]

29. \((x + 2)^2 = 14\)

\[x + 2 = \pm \sqrt{14}\]

\[x = -2 \pm \sqrt{14}\]

30. \((x - 5)^2 = 30\)

\[x - 5 = \pm \sqrt{30}\]

\[x = 5 \pm \sqrt{30}\]

31. \((2x - 1)^2 = 18\)

\[2x - 1 = \pm \sqrt{18}\]

\[2x = 1 \pm 3\sqrt{2}\]

\[x = \frac{1 \pm 3\sqrt{2}}{2}\]

32. \((4x + 7)^2 = 44\)

\[4x + 7 = \pm \sqrt{44}\]

\[4x = -7 \pm 2\sqrt{11}\]

\[x = \frac{-7 \pm 2\sqrt{11}}{4}\]

33. \((x - 7)^2 = (x + 3)^2\)

\[x - 7 = \pm (x + 3)\]

\[x - 7 = x + 3 \text{ or } x - 7 = -x - 3\]

\[-7 \neq 3 \text{ or } 2x = 4\]

\[x = 2\]

The only solution to the equation is \(x = 2\).

34. \((x + 5)^2 = (x + 4)^2\)

\[x + 5 = \pm (x + 4)\]

\[x + 5 = +(x + 4) \text{ or } x + 5 = -(x + 4)\]

\[5 \neq 4 \text{ or } x + 5 = -x - 4\]

\[2x = -9\]

\[x = \frac{-9}{2}\]

The only solution to the equation is \(x = \frac{-9}{2}\).

35. \(x^2 + 4x - 32 = 0\)

\[x^2 + 4x = 32\]

\[x^2 + 4x + 2^2 = 32 + 2^2\]

\[(x + 2)^2 = 36\]

\[x + 2 = \pm 6\]

\[x = -2 \pm 6\]

\[x = 4 \text{ or } x = -8\]

36. \(x^2 - 2x - 3 = 0\)

\[x^2 - 2x = 3\]

\[x^2 - 2x + (-1)^2 = 3 + (-1)^2\]

\[(x - 1)^2 = 4\]

\[x - 1 = \pm \sqrt{4}\]

\[x = 1 \pm 2\]

\[x = 3 \text{ or } x = -1\]

37. \(x^2 + 12x + 25 = 0\)

\[x^2 + 12x = -25\]

\[x^2 + 12x + 6^2 = -25 + 6^2\]

\[(x + 6)^2 = 11\]

\[x + 6 = \pm \sqrt{11}\]

\[x = -6 \pm \sqrt{11}\]
38. \(x^2 + 8x + 14 = 0\)
   \[x^2 + 8x = -14\]
   \[x^2 + 8x + 4 = -14 + 16\]
   \[(x + 4)^2 = 2\]
   \[x + 4 = \pm \sqrt{2}\]
   \[x = -4 \pm \sqrt{2}\]

39. \(9x^2 - 18x = -3\)
   \[x^2 - 2x = -\frac{1}{3}\]
   \[x^2 - 2x + 1^2 = -\frac{1}{3} + 1^2\]
   \[x^2 - 2x + \left(\frac{2}{3}\right)^2 = \frac{14}{9} + \frac{4}{9}\]
   \[x - 1 = \pm \frac{\sqrt{2}}{3}\]
   \[x = 1 \pm \frac{\sqrt{6}}{3}\]

40. \(9x^2 - 12x = 14\)
   \[x^2 - \frac{4}{3}x = \frac{14}{9}\]
   \[x^2 - 2x + 1^2 = -\frac{1}{3} + 1^2\]
   \[\left(x - \frac{2}{3}\right)^2 = \frac{18}{9}\]
   \[x = \frac{2}{3} \pm \frac{\sqrt{2}}{3}\]

41. \(8 + 4x - x^2 = 0\)
   \[-x^2 + 4x + 8 = 0\]
   \[x^2 - 4x - 8 = 0\]
   \[x^2 - 4x = 8\]
   \[x^2 - 4x + 2^2 = 8 + 2^2\]
   \[(x - 2)^2 = 12\]
   \[x - 2 = \pm \sqrt{12}\]
   \[x = 2 \pm 2\sqrt{3}\]

42. \(-x^2 + x - 1 = 0\)
   \[x^2 - x + 1 = 0\]
   \[x^2 - x + \frac{1}{4} = -1 + \frac{1}{4}\]
   \[(x - \frac{1}{2})^2 = -\frac{3}{4}\]
   No real solution

43. \(2x^2 + 5x - 8 = 0\)
   \[2x^2 + 5x = 8\]
   \[x^2 + \frac{5}{2}x = 4\]
   \[x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = 4 + \left(\frac{5}{4}\right)^2\]
   \[\left(x + \frac{5}{4}\right)^2 = \frac{89}{16}\]
   \[x + \frac{5}{4} = \pm \frac{\sqrt{89}}{4}\]
   \[x = -\frac{5}{4} \pm \frac{\sqrt{89}}{4}\]
   \[x = -\frac{5 \pm \sqrt{89}}{4}\]

44. \(4x^2 - 4x - 99 = 0\)
   \[x^2 - x = \frac{99}{4}\]
   \[x^2 - x + \left(-\frac{1}{2}\right)^2 = \frac{99}{4} + \frac{1}{4}\]
   \[\left(x - \frac{1}{2}\right)^2 = \frac{100}{4}\]
   \[\left(x - \frac{1}{2}\right)^2 = 25\]
   \[x - \frac{1}{2} = \pm \sqrt{25}\]
   \[x = \frac{1}{2} \pm 5 = \frac{11}{2}, \frac{9}{2}\]

45. \(\frac{1}{x^2 + 2x + 5} = \frac{1}{x^2 + 2x + 1^2 - 1^2 + 5}\)
   \[= \frac{1}{(x + 1)^2 + 4}\]

46. \(\frac{1}{x^2 - 12x + 19} = \frac{1}{(x^2 - 12x + (-6)^2) + 19 - 36}\)
   \[= \frac{1}{(x - 6)^2 - 17}\]
47. \[ \frac{4}{x^2 + 4x - 3} = \frac{4}{x^2 + 4x + 4 - 4 - 3} = \frac{4}{(x+2)^2 - 7} \]

49. \[ \frac{1}{\sqrt{6x-x^2}} = \frac{1}{\sqrt{-1(x^2 - 6x + 3^2 - 3^2)}} = \frac{1}{\sqrt{-1[(x-3)^2 - 9]}} = \frac{1}{\sqrt{-(x-3)^2 + 9}} = \frac{1}{\sqrt{9 - (x-3)^2}} \]

51. (a) \( y = (x + 3)^2 - 4 \)

(b) The \( x \)-intercepts are \((-1, 0)\) and \((-5, 0)\).

52. (a) \( y = (x - 4)^2 - 1 \)

(b) The \( x \)-intercepts are \((5, 0)\) and \((3, 0)\).

53. (a) \( y = 1 - (x - 2)^2 \)

(b) The \( x \)-intercepts are \((1, 0)\) and \((3, 0)\).

54. (a) \( y = 9 - (x - 8)^2 \)

(b) The \( x \)-intercepts are \((5, 0)\) and \((11, 0)\).
55. (a) \( y = -4x^2 + 4x + 3 \)

(b) The x-intercepts are \((-\frac{1}{2}, 0)\) and \((\frac{3}{2}, 0)\).

(d) The x-intercepts of the graph are solutions to the equation \(0 = -4x^2 + 4x + 3\).

(c) \( 0 = -4x^2 + 4x + 3 \)
\[
4x^2 - 4x = 3
\]
\[
4(x^2 - x) = 3
\]
\[
x^2 - x = \frac{3}{4}
\]
\[
x^2 - x + (\frac{1}{2})^2 = \frac{3}{4} + (\frac{1}{2})^2
\]
\[
(x - \frac{1}{2})^2 = 1
\]
\[
x = \frac{1}{2} \pm 1
\]
\[
x = \frac{3}{2} \text{ or } x = -\frac{1}{2}
\]

56. (a) \( y = 4x^2 - 1 \)

(b) The x-intercepts are \((-\frac{1}{2}, 0)\) and \((\frac{1}{2}, 0)\).

(c) \( 0 = 4x^2 - 1 \)
\[
4x^2 = 1
\]
\[
x^2 = \frac{1}{4}
\]
\[
x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}
\]

(d) The x-intercepts of the graph are solutions to the equation \(0 = 4x^2 - 1\).

57. (a) \( y = x^2 + 3x - 4 \)

(b) The x-intercepts are \((-4, 0)\) and \((1, 0)\).

(c) \( 0 = x^2 + 3x - 4 \)
\[
0 = (x + 4)(x - 1)
\]
\[
x + 4 = 0 \text{ or } x - 1 = 0
\]
\[
x = -4 \text{ or } x = 1
\]

(d) The x-intercepts of the graph are solutions to the equation \(0 = x^2 + 3x - 4\).

58. (a) \( y = x^2 - 5x - 24 \)

(b) The x-intercepts are \((8, 0)\) and \((-3, 0)\).

(c) \( 0 = x^2 - 5x - 24 \)
\[
(x - 8)(x + 3) = 0
\]
\[
x - 8 = 0 \Rightarrow x = 8
\]
\[
x + 3 = 0 \Rightarrow x = -3
\]

(d) The x-intercepts of the graph are solutions to the equation \(0 = x^2 - 5x - 24\).

59. \( 2x^2 - 5x + 5 = 0 \)
\[
b^2 - 4ac = (-5)^2 - 4(2)(5) = -15 < 0
\]
No real solution

60. \(-5x^2 - 4x + 1 = 0 \)
\[
b^2 - 4ac = (-4)^2 - 4(-5)(1)
\]
\[
= 16 + 20 = 36 > 0
\]
Two real solutions

61. \( 2x^2 - x - 1 = 0 \)
\[
b^2 - 4ac = (-1)^2 - 4(2)(-1) = 9 > 0
\]
Two real solutions

62. \( x^2 - 4x + 4 = 0 \)
\[
b^2 - 4ac = (-4)^2 - 4(1)(4)
\]
\[
= 16 - 16 = 0
\]
One repeated solution
63. \( \frac{1}{5}x^2 - 5x + 25 = 0 \)

\[ b^2 - 4ac = (-5)^2 - 4\left(\frac{1}{5}\right)(25) = -\frac{25}{5} < 0 \]

No real solution

64. \( \frac{4}{5}x^2 - 8x + 28 = 0 \)

\[ b^2 - 4ac = (-8)^2 - 4\left(\frac{4}{5}\right)(28) = 64 - 64 = 0 \]

One repeated solution

65. \( 0.2x^2 + 1.2x - 8 = 0 \)

\[ b^2 - 4ac = (1.2)^2 - 4(0.2)(-8) = 7.84 > 0 \]

Two real solutions

66. \( 9 + 2.4x - 8.3x^2 = 0 \)

\[ b^2 - 4ac = (2.4)^2 - 4(-8.3)(9) = 5.76 + 298.8 = 304.56 > 0 \]

Two real solutions

67. \( 2x^2 + x - 1 = 0 \)

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} \\
&= \frac{-1 \pm \sqrt{1 + 8}}{4} \\
&= \frac{-1 \pm 3}{4} = 1, -\frac{1}{2}
\end{align*}
\]

68. \( 2x^2 - x - 1 = 0 \)

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{(1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} \\
&= \frac{1 \pm \sqrt{1 + 8}}{4} \\
&= \frac{1 \pm 3}{4} = 1, \frac{1}{2}
\end{align*}
\]

69. \( 16x^2 + 8x - 3 = 0 \)

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-8 \pm \sqrt{8^2 - 4(16)(-3)}}{2(16)} \\
&= \frac{-8 \pm 16}{32} = \frac{1}{4}, \frac{3}{4}
\end{align*}
\]

70. \( 25x^2 - 20x + 3 = 0 \)

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(3)}}{2(25)} \\
&= \frac{20 \pm \sqrt{400 - 300}}{50} \\
&= \frac{20 \pm 10 \sqrt{5}}{50} = \frac{3 \pm \sqrt{5}}{5}
\end{align*}
\]

71. \( 2 + 2x - x^2 = 0 \)

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)} \\
&= \frac{-2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}
\end{align*}
\]

72. \( x^2 - 10x + 22 = 0 \)

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)} \\
&= \frac{10 \pm \sqrt{100 - 88}}{2} = 5 \pm \sqrt{3}
\end{align*}
\]

73. \( x^2 + 14x + 44 = 0 \)

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-14 \pm \sqrt{14^2 - 4(1)(44)}}{2(1)} \\
&= \frac{-14 \pm 2\sqrt{5}}{2} = -7 \pm \sqrt{5}
\end{align*}
\]

74. \( 6x = 4 - x^2 \)

\[
\begin{align*}
x^2 + 6x - 4 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-6 \pm \sqrt{6^2 - 4(1)(-4)}}{2(1)} \\
&= \frac{-6 \pm \sqrt{36 + 16}}{2} = \frac{-6 \pm 2\sqrt{13}}{2} = -3 \pm \sqrt{13}
\end{align*}
\]

75. \( x^2 + 8x - 4 = 0 \)

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)} \\
&= \frac{-8 \pm 4\sqrt{5}}{2} = -4 \pm 2\sqrt{5}
\end{align*}
\]
76. \(4x^2 - 4x - 4 = 0\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-4)}}{2(1)}
\]

\[
x = \frac{4 \pm \sqrt{16 + 16}}{2}
\]

\[
x = \frac{4 \pm 4}{2}
\]

\[
x = 2, -2
\]

77. \(12x - 9x^2 = -3\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-9 \pm \sqrt{(-9)^2 - 4(1)(-3)}}{2(1)}
\]

\[
x = \frac{-9 \pm \sqrt{81 + 12}}{2}
\]

\[
x = \frac{-9 \pm 9}{2}
\]

\[
x = -3, 3
\]

78. \(16x^2 + 22 = 40x\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-22 \pm \sqrt{(-22)^2 - 4(16)(40)}}{2(16)}
\]

\[
x = \frac{-22 \pm \sqrt{484 - 2560}}{32}
\]

\[
x = \frac{-22 \pm \sqrt{-2076}}{32}
\]

\[
x = \frac{-22 \pm 45.2}{32}
\]

\[
x = -0.7, 1.4
\]

79. \(9x^2 + 24x + 16 = 0\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-24 \pm \sqrt{24^2 - 4(1)(16)}}{2(1)}
\]

\[
x = \frac{-24 \pm \sqrt{576 - 64}}{2}
\]

\[
x = \frac{-24 \pm 24}{2}
\]

\[
x = -8, 0
\]

80. \(36x^2 + 24x - 7 = 0\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-24 \pm \sqrt{24^2 - 4(36)(-7)}}{2(36)}
\]

\[
x = \frac{-24 \pm \sqrt{576 + 1008}}{72}
\]

\[
x = \frac{-24 \pm \sqrt{1584}}{72}
\]

\[
x = \frac{-24 \pm 32}{72}
\]

\[
x = \frac{-32}{72}, 0
\]

81. \(4x^2 + 4x = 7\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(4)(-7)}}{2(4)}
\]

\[
x = \frac{-4 \pm 8\sqrt{2}}{8}
\]

\[
x = \frac{1}{2} \pm \frac{\sqrt{2}}{2}
\]

82. \(16x^2 - 40x + 5 = 0\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(1)(5)}}{2(16)}
\]

\[
x = \frac{40 \pm \sqrt{1600 - 20}}{32}
\]

\[
x = \frac{40 \pm 39}{32}
\]

\[
x = \frac{39}{16}, \frac{1}{2}
\]

83. \(28x = 49x^2\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-28 \pm \sqrt{28^2 - 4(-49)(4)}}{2(-49)}
\]

\[
x = \frac{-28 \pm 14}{98}
\]

\[
x = \frac{-1}{7}, 1
\]

84. \(3x + x^2 - 1 = 0\)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-1 \pm \sqrt{1 + 4(1)(1)}}{2(1)}
\]

\[
x = \frac{-1 \pm 2}{2}
\]

\[
x = -1, 1
\]

85. \(-2r^2 + 8r - 5 = 0\)

\[
t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
t = \frac{-8 \pm \sqrt{8^2 - 4(-2)(-5)}}{2(-2)}
\]

\[
t = \frac{-8 \pm 2\sqrt{6}}{-4}
\]

\[
t = \frac{-8 \pm 2\sqrt{6}}{-4}
\]

\[
t = \frac{4 \pm \sqrt{6}}{2}
\]

86. \(25h^2 + 80h + 61 = 0\)

\[
h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
h = \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)}
\]

\[
h = \frac{-80 \pm \sqrt{6400 - 6100}}{50}
\]

\[
h = \frac{-80 \pm 10\sqrt{3}}{50}
\]

\[
h = \frac{-8 \pm \sqrt{3}}{5}
\]

87. \((y - 5)^2 = 2y\)

\[
y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
y = \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}
\]

\[
y = \frac{-2 \pm \sqrt{4 - 8}}{2}
\]

\[
y = \frac{-2 \pm 2\sqrt{2}}{2}
\]

\[
y = 1 \pm \sqrt{2}
\]
88. \[(z + 6)^2 = -2z\]
\[z^2 + 12z + 36 = -2z\]
\[z^2 + 14z + 36 = 0\]
\[z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[= \frac{-14 \pm \sqrt{196 - 4(1)(36)}}{2(1)}\]
\[= \frac{-14 \pm \sqrt{52}}{2}\]
\[= -7 \pm \sqrt{13}\]

90. \[
\left(\frac{5}{7}x - 14\right)^2 = 8x
\]
\[
\frac{25}{49}x^2 - \frac{140}{7}x + 196 = 8x
\]
\[
\frac{25}{49}x^2 - 20x + 196 = 0
\]
\[
25x^2 - 1372x + 9604 = 0
\]
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[= \frac{-(-1372) \pm \sqrt{(-1372)^2 - 4(25)(9604)}}{2(25)}\]
\[= \frac{1372 \pm \sqrt{921984}}{50}\]
\[= \frac{686 \pm 196\sqrt{6}}{25}\]

92. \[2x^2 - 2.50x - 0.42 = 0\]
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[= \frac{-(-2.50) \pm \sqrt{(-2.50)^2 - 4(2)(-0.42)}}{2(2)}\]
\[= \frac{2.50 \pm \sqrt{9.61}}{4}\]
\[= 1.400, -0.150\]

94. \[-0.005x^2 + 0.101x - 0.193 = 0\]
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[= \frac{-0.101 \pm \sqrt{0.101^2 - 4(-0.005)(-0.193)}}{2(-0.005)}\]
\[= \frac{-0.101 \pm \sqrt{0.006341}}{-0.01}\]
\[\approx 2.137, 18.063\]

95. \[422x^2 - 506x - 347 = 0\]
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[= \frac{-(-506) \pm \sqrt{(-506)^2 - 4(422)(-347)}}{2(422)}\]
\[\approx 1.687, -0.488\]
96. \(1100x^2 + 326x - 715 = 0\)
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
= \frac{-326 \pm \sqrt{(326)^2 - 4(1100)(-715)}}{2(1100)}
\]
\[
= \frac{-326 \pm \sqrt{3252.276}}{2200} = 0.672, -0.968
\]

97. \(12.67x^2 + 31.55x + 8.09 = 0\)
\[
x = \frac{-31.55 \pm \sqrt{(31.55)^2 - 4(12.67)(8.09)}}{2(12.67)}
\]
\[
x = -2.200, -0.290
\]

98. \(-3.22x^2 - 0.08x + 28.651 = 0\)
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
= \frac{-(-0.08) \pm \sqrt{(-0.08)^2 - 4(-3.22)(28.651)}}{2(-3.22)}
\]
\[
= \frac{0.08 \pm \sqrt{369.031}}{-6.44} = -2.995, 2.971
\]

100. \(11x^2 + 33x = 0\) Factor.
\[
11(x^2 + 3x) = 0
\]
\[
x(x + 3) = 0
\]
\[
x = 0 \text{ or } x + 3 = 0
\]
\[
x = 0 \text{ or } x = -3
\]

102. \(x^2 - 14x + 49 = 0\) Extract square roots.
\[
(x - 7)^2 = 0
\]
\[
x - 7 = 0
\]
\[
x = 7
\]

104. \(x^2 + 3x - \frac{3}{4} = 0\) Complete the square.
\[
x^2 + 3x + (\frac{3}{2})^2 = \frac{3}{4} + \frac{9}{4}
\]
\[
(x + \frac{3}{2})^2 = 3
\]
\[
x + \frac{3}{2} = \pm \sqrt{3}
\]
\[
x = -\frac{3}{2} \pm \sqrt{3}
\]

106. \(a^2x^2 - b^2 = 0\) Factor.
\[
(ax + b)(ax - b) = 0
\]
\[
ax + b = 0 \Rightarrow x = -\frac{b}{a}
\]
\[
ax - b = 0 \Rightarrow x = \frac{b}{a}
\]

101. \((x + 3)^2 = 81\) Extract square roots.
\[
x + 3 = \pm 9
\]
\[
x + 3 = 9 \text{ or } x + 3 = -9
\]
\[
x = 6 \text{ or } x = -12
\]

103. \(x^2 - x - \frac{11}{4} = 0\) Complete the square.
\[
x^2 - x + (\frac{1}{2})^2 = \frac{1}{4} + (\frac{1}{2})^2
\]
\[
(x - \frac{1}{2})^2 = \frac{12}{4}
\]
\[
x - \frac{1}{2} = \pm \sqrt{3}
\]
\[
x = \frac{1}{2} \pm \sqrt{3}
\]

105. \((x + 1)^2 = x^2\) Extract square roots.
\[
x^2 = (x + 1)^2
\]
\[
x = \pm (x + 1)
\]
For \(x = +(x + 1):\)
\[
0 \neq 1 \text{ No solution}
\]
For \(x = -(x + 1):\)
\[
2x = -1
\]
\[
x = -\frac{1}{2}
\]
107. \(3x + 4 = 2x^2 - 7\)  
**Quadratic Formula**

\[0 = 2x^2 - 3x - 11\]

\[x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)}\]

\[= \frac{3 \pm \sqrt{97}}{4}\]

\[= \frac{3}{4} \pm \frac{\sqrt{97}}{4}\]

108. \(4x^2 + 2x + 4 = 2x + 8\)  
**Factor.**

\[4x^2 - 4 = 0\]

\[4(x^2 - 1) = 0\]

\[(x + 1)(x - 1) = 0\]

\[x + 1 = 0\] or \(x - 1 = 0\)

\[x = -1\] \(x = 1\)

109. (a) 

(b) \(w(w + 14) = 1632\)

(c) \(w^2 + 14w - 1632 = 0\)

\((w + 48)(w - 34) = 0\)

\(w = -48\) or \(w = 34\)

Since \(w\) must be greater than zero, we have \(w = 34\) feet and the length is \(w + 14 = 48\) feet.

110. Total fencing: \(4x + 3y = 100\)

Total area: \(2xy = 350\)

\[x = \frac{100 - 3y}{4}\]

\[2\left(\frac{100 - 3y}{4}\right)y = 350\]

\[
\frac{1}{2}(100y - 3y^2) - 350 = 0
\]

\[-3y^2 + 100y - 700 = 0
\]

\((3y - 70)(-y + 10) = 0\)

\[3y - 70 = 0 \Rightarrow y = \frac{70}{3}\]

\[-y + 10 = 0 \Rightarrow y = 10\]

For \(y = \frac{70}{3}\),

For \(y = 10:\)

\[2x\left(\frac{70}{3}\right) = 350\]

\[x = 7.5\]

There are two solutions: \(x = 7.5\) meters and \(y = \frac{70}{3}\) meters or \(x = 17.5\) meters and \(y = 10\) meters.

111. \(S = x^2 + 4sh\)

\[84 = x^2 + 4x(2)\]

\[0 = x^2 + 8x - 84\]

\[0 = (x + 14)(x - 6)\]

\[x = -14\] or \(x = 6\)

Since \(x\) must be positive, we have \(x = 6\) inches. The dimensions of the box are 6 inches \(\times\) 6 inches \(\times\) 2 inches.

112. Volume: \(2x^2 = 200\)

\[x^2 = 100\]

\[x = \pm 10\]

Original size of material:

Side: \(x + 4 = 10 + 4 = 14\) centimeters

Dimensions: 14 centimeters \(\times\) 14 centimeters

113. \((200 - 2x)(100 - 2x) = \frac{1}{2}(100)(200)\)

\[20,000 - 600x + 4x^2 = 10,000\]

\[4x^2 - 600x + 10,000 = 0\]

\[4(x^2 - 150x + 2500) = 0\]

Thus, \(a = 1, b = -150,\) and \(c = 2500.\)

—CONTINUED—
113. —CONTINUED—

\[
x = \frac{150 \pm \sqrt{(150)^2 - 4(1)(2500)}}{2(1)} \approx \frac{150 \pm 111.8034}{2}
\]

\[
x \approx \frac{150 + 111.8034}{2} \approx 130.902 \text{ feet, not possible since the lot is only 100 feet wide.}
\]

\[
x \approx \frac{150 - 111.8034}{2} \approx 19.098 \text{ feet}
\]

The person must go around the lot \(\frac{19.098 \text{ feet}}{24 \text{ inches}} = \frac{19.098 \text{ feet}}{2 \text{ feet}} \approx 9.5 \text{ times.}\)

114. Original arrangement: \(x\) rows, \(y\) seats per row, \(xy = 72\).

\[y = \frac{72}{x}\]

New arrangement: \((x - 2)\) rows, \((y + 3)\) seats per row

\((x - 2)(y + 3) = 72\)

\((x - 2)\left(\frac{72}{x} + 3\right) = 72\)

\(x(x - 2)\left(\frac{72}{x} + 3\right) = 72x\)

\(72x + 3x^2 - 144 - 6x = 72x\)

\(3x^2 - 6x - 144 = 0\)

\(x^2 - 2x - 48 = 0\)

\((x - 8)(x + 6) = 0\)

\(x - 8 = 0 \implies x = 8\)

\(x + 6 = 0 \implies x = -6\)

Originally, there were 8 rows of seats with \(\frac{72}{8} = 9\) seats per row.

115. \(s = -16t^2 + 32,000\)

(a) \(-16t^2 + 32,000 = 0\)

\[t^2 = 2000\]

\[t = \sqrt{2000} = 20\sqrt{5} \text{ seconds}\]

\(\approx 44.72 \text{ seconds}\)

(b) Model: \((\text{Rate}) \cdot (\text{time}) = (\text{distance})\)

Labels: \(\text{Rate} = 500 \text{ miles per hour}\)

\(\text{Time} = \frac{20\sqrt{5}}{3600} \approx 0.0124 \text{ hour}\)

\(\text{Distance} = d\)

Equation: \(d = 500(0.0124) \approx 6.2 \text{ miles}\)

The bomb will travel approximately 6.2 miles horizontally.

116. (a) \(s = -16t^2 + v_0t + s_0\)

Since the object was dropped, \(v_0 = 0\), and the initial height is \(s_0 = 984\). Thus, \(s = -16t^2 + 984\).

(b) \(s = -16(4)^2 + 984 = 728 \text{ feet}\)

(c) \(0 = -16(4)^2 + 984\)

\[16t^2 = 984\]

\[t^2 = \frac{984}{16}\]

\[t = \sqrt{\frac{984}{16}} = \sqrt{246} \approx 7.84\]

It will take the coin about 7.84 seconds to strike the ground.

117. \(s = -16t^2 + v_0t + s_0\)

(a) \(v_0 = 100 \text{ mph} = \frac{(100)(5280)}{3600} = 146.67 \text{ ft/sec}\)

\[s_0 = 6\frac{1}{2} \text{ feet}\]

\(s = -16t^2 + 146\frac{2}{3}t + 6\frac{1}{2}\)

(b) When \(t = 3\): \(s(3) = 302.25 \text{ feet}\)

When \(t = 4\): \(s(4) = 336.92 \text{ feet}\)

When \(t = 5\): \(s(5) = 339.58 \text{ feet}\)

During the interval \(3 \leq t \leq 5\), the baseball’s speed decreased due to gravity.

(c) The ball hits the ground when \(s = 0\).

\[-16t^2 + 146\frac{2}{3}t + 6\frac{1}{2} = 0\]

By the Quadratic Formula, \(t \approx -0.042\) or \(t \approx 9.209\).

Assuming that the ball is not caught and drops to the ground, it will be in the air for approximately 9.209 seconds.
118. (a) \[ s = -16t^2 + v_0t + s_0 \]

Since the object was dropped, \( v_0 = 0 \), and the initial height is \( s_0 = 1815 \). Thus, \( s = -16t^2 + 1815 \).

(b) The object reaches the ground between \( t = 10 \) seconds and \( t = 12 \) seconds. Numerical approximations will vary, though 10.7 seconds is a reasonable estimate.

(d) \[ 0 = -16t^2 + 1815 \]

\[ 16t^2 = 1815 \]

\[ t^2 = \frac{1815}{16} \]

\[ t = \sqrt{\frac{1815}{16}} = \frac{11\sqrt{15}}{4} \approx 10.65 \]

It will take the object about 10.65 seconds to reach the ground.

(c) The zero of the graph is at \( t = 10.65 \) seconds.

119. \( P = -0.0081t^2 + 0.417t + 1.99, \ 7 \leq t \leq 13 \)

(a) The average admission price reached or surpassed $5.00 in 1999.

(b) \[ -0.0081t^2 + 0.417t + 1.99 = 5.00 \]

\[ -0.0081t^2 + 0.417t - 3.01 = 0 \]

By the Quadratic Formula, \( t \approx 8.68 \) or 42.80. Since \( 7 \leq t \leq 13 \), we choose \( t \approx 8.68 \approx 9 \) which corresponds to 1999.

(c) For 2008, let \( t = 18 \): \( P(18) \approx 6.87 \). Answers will vary.

120. (a) The median income reached and surpassed $40,000 in 1998, \( x = 8.65 \).

(b) Substituting \( x = 8.65 \) into the model gives $40,003.46. Substituting \( x = 8.6 \) into the model gives $39,940.31.

(c) For 2008, \( t = 18 \) and \( I = 40,108.68 \).
   For 2013, \( t = 23 \) and \( I = 30,609.28 \).
   No. The model shows the median income decreasing after 2002.
121. \( x^2 + x^2 = s^2 \)

\[
2x^2 = 25 \\
x^2 = \frac{25}{2} \\
x = \sqrt{\frac{25}{2}} \\
= \frac{5}{\sqrt{2}} \\
= \frac{5\sqrt{2}}{2} \approx 3.54 \text{ centimeters}
\]

Each leg in the right triangle is approximately 3.54 centimeters.

122. **Model:** \((\text{height})^2 + (\text{half of side})^2 = (\text{side})^2\)

**Labels:** height = 10 inches, side = \(s\), half of side = \(\frac{s}{2}\)

**Equation:**

\[
10^2 + \left(\frac{s}{2}\right)^2 = s^2 \\
100 + \frac{s^2}{4} = s^2 \\
\frac{3}{4}s^2 = 100 \\
\frac{s^2}{3} = \frac{100}{3} \\
s = \sqrt{\frac{400}{3}} = \frac{20\sqrt{3}}{3} \approx 11.55 \text{ inches}
\]

Each side of the equilateral triangle is approximately 11.55 inches long.

123.

\[
d_N = (3 \text{ hours})(r + 50 \text{ mph}) \\
d_E = (3 \text{ hours})(r \text{ mph}) \\
d_N^2 + d_E^2 = 2440^2 \\
9(r + 50)^2 + 9r^2 = 2440^2 \\
18r^2 + 900r - 5,931,100 = 0
\]

\[
r = \frac{-900 \pm \sqrt{900^2 - 4(18)(-5,931,100)}}{2(18)} \approx \frac{-900 \pm 60\sqrt{118,847}}{36}
\]

Using the positive value for \(r\), we have one plane moving northbound at \(r + 50 \approx 600 \text{ miles per hour}\) and one plane moving eastbound at \(r \approx 550 \text{ miles per hour}\).

124. (a) **Model:** \((\text{winch})^2 + (\text{distance to dock})^2 = (\text{length of rope})^2\)

**Labels:** winch = 15, distance to dock = \(x\), length of rope = \(l\)

**Equation:** \(15^2 + x^2 = l^2\)

(b) When \(l = 75\):

\[
x^2 = 5625 - 225 = 5400 \\
x = \sqrt{5400} = 30\sqrt{6} \approx 73.5 \text{ feet}
\]

The boat is approximately 73.5 feet from the dock when there is 75 feet of rope out.
125. \( x(20 - 0.0002x) = 500,000 \)
\[
0 = 0.0002x^2 - 20x + 500,000
\]
\[
0 = x^2 - 100,000x + 2,500,000,000
\]
\[
0 = (x - 50,000)^2
\]
\[
x = 50,000 \text{ units}
\]

126. \( x(60 - 0.0004x) = 220,000 \)
\[
60x - 0.0004x^2 - 220,000 = 0
\]
\[
-4x^2 + 600,000x - 2,200,000,000 = 0
\]
\[
x^2 - 150,000x + 550,000,000 = 0
\]
\[
x = \frac{150,000 \pm \sqrt{150,000^2 - 4(550,000,000)}}{2} = 3761 \text{ units or } 146,239 \text{ units}
\]

127. \( 0.125x^2 + 20x + 500 = 14,000 \)
\[
0.125x^2 + 20x - 13,500 = 0
\]
\[
x = \frac{-20 \pm \sqrt{20^2 - 4(0.125)(-13,500)}}{2(0.125)}
\]
Using the positive value for \( x \), we have
\[
x = \frac{-20 + \sqrt{7150}}{0.25} = 258 \text{ units.}
\]

129. \( 800 + 0.04x + 0.002x^2 = 1680 \)
\[
0.002x^2 + 0.04x - 880 = 0
\]
\[
x = \frac{-0.04 \pm \sqrt{(0.04)^2 - 4(0.002)(-880)}}{2(0.002)}
\]
\[
x = \frac{-0.04 \pm \sqrt{7.0416}}{0.004}
\]
Choosing the positive value for \( x \), we have
\[
x = \frac{-0.04 + \sqrt{7.0416}}{0.004} = 653 \text{ units.}
\]

131. \( M = 1.835t^2 + 3.58t + 333.0, \ 5 \leq t \leq 13 \)

(a)

<table>
<thead>
<tr>
<th>( t )</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M ) (in billions)</td>
<td>$396.78</td>
<td>$420.54</td>
<td>$447.98</td>
<td>$479.08</td>
<td>$513.86</td>
<td>$552.30</td>
<td>$594.42</td>
<td>$640.20</td>
<td>$689.66</td>
</tr>
</tbody>
</table>

The total money in circulation reached or surpassed $600 billion in 2001.

(b) \( 1.835t^2 + 3.58t + 333.0 = 600 \)
\[
1.835t^2 + 3.58t - 267.0 = 0
\]

By the Quadratic Formula, \( t = 11.1 \text{ or } -13.1 \). Since \( 5 \leq t \leq 13 \), we choose \( t = 11.1 \) which corresponds to 2001.

(c) For 2008, let \( t = 18 \): \( M(18) = \$991.98 \text{ billion} \)
Answers will vary.
132. (a) \[150 = 0.45x^2 - 1.65x + 50.75\]
\[0 = 0.45x^2 - 1.65x - 99.25\]
\[x = \frac{1.65 \pm \sqrt{(-1.65)^2 - 4(0.45)(-99.25)}}{2(0.45)}\]
\[= -13.1, 16.8\]
Because \(10 \leq x \leq 25\), choose 16.8°C.

(b) \[x = 10: \quad 0.45(10)^2 - 1.65(10) + 50.75 = 79.25\]
\[x = 20: \quad 0.45(20)^2 - 1.65(20) + 50.75 = 197.75\]
\[197.75 \div 79.25 = 2.5\]
Oxygen consumption is increased by a factor of approximately 2.5.

133. The other two distances are 354.5 miles and 433.5 miles.

134. (a) \[V = 1024 \text{ ft}^3, \text{ so:}\]
\[4x(x + 1) = 1024\]
\[4x^2 + 4x - 1024 = 0\]
\[x = 15.5078 \text{ ft}\]
\[x + 1 = 16.5078 \text{ ft}\]

(b) 1 ft³ weighs approximately 62.4 lbs., so
\[1024 \text{ ft}^3 \times 62.4 \text{ lbs/ft}^3 = 63,897.6 \text{ lbs.}\]
(c) \[\frac{1024 \text{ ft}^3}{0.13368 \text{ ft}^3/\text{gal}} = 7660.08 \text{ gal}\]
At 5 gallons per minute:
\[\frac{7660.08 \text{ gal}}{5 \text{ gals/min}} = 1532.02 \text{ min} = 25.5 \text{ hours}\]

135. False
\[b^2 - 4ac = (-1)^2 - 4(-3)(-10) < 0,\]
so, the quadratic equation has no real solutions.

136. False. The product must equal zero for the Zero-Factor property to be used.

137. The student should have subtracted 15x from both sides so that the equation is equal to zero. By factoring out an \(x\), there are two solutions, \(x = 0\) and \(x = 6\).

138. (a) Neither. A linear equation has at most one real solution, and a quadratic equation has at most two real solutions.
(b) Both
(c) Quadratic
(d) Neither
139. \(3(x + 4)^2 + (x + 4) - 2 = 0\)

(a) Let \(u = x + 4\)

\[3u^2 + u - 2 = 0\]

\((3u - 2)(u + 1) = 0\)

\(3u - 2 = 0\) or \(u + 1 = 0\)

\(u = \frac{2}{3}\) or \(u = -1\)

\(x + 4 = \frac{2}{3}\) or \(x + 4 = -1\)

\(x = -\frac{10}{3}\) or \(x = -5\)

(b) \(3(x^2 + 8x + 16) + (x + 4) - 2 = 0\)

\(3x^2 + 24x + 48 + x + 4 - 2 = 0\)

\(3x^2 + 25x + 50 = 0\)

\((3x + 10)(x + 5) = 0\)

\(3x + 10 = 0\) or \(x + 5 = 0\)

\(x = -\frac{10}{3}\) or \(x = -5\)

(c) The method of part (a) reduces the number of algebraic steps.

140. (a) \(ax^2 + bx = 0\)

\(x(ax + b) = 0\)

\(x = 0\)

\(ax + b = 0 \Rightarrow x = -\frac{b}{a}\)

(b) \(ax^2 - ax = 0\)

\(ax(x - 1) = 0\)

\(ax = 0 \Rightarrow x = 0\)

\(x - 1 = 0 \Rightarrow x = 1\)

141. \(-3\) and 6

One possible equation is:

\((x - (-3))(x - 6) = 0\)

\((x + 3)(x - 6) = 0\)

\(x^2 - 3x - 18 = 0\)

Any non-zero multiple of this equation would also have these solutions.

142. \((x - (-4))(x - (-11)) = 0\)

\((x + 4)(x + 11) = 0\)

\(x^2 + 15x + 44 = 0\)

143. \(8\) and 14

One possible equation is:

\((x - 8)(x - 14) = 0\)

\(x^2 - 22x + 112 = 0\)

Any non-zero multiple of this equation would also have these solutions.

144. \(x = \frac{1}{5} \Rightarrow 6x = 1 \Rightarrow 6x - 1\) is a factor.

\(x = -\frac{5}{2} \Rightarrow 5x = -2 \Rightarrow 5x + 2\) is a factor.

\((6x - 1)(5x + 2) = 0\)

\(30x^2 + 7x - 2 = 0\)

145. \(1 + \sqrt{2}\) and \(1 - \sqrt{2}\)

One possible equation is:

\([x - (1 + \sqrt{2})][x - (1 - \sqrt{2})] = 0\)

\([(x - 1) - \sqrt{2}][(x - 1) + \sqrt{2}] = 0\)

\((x - 1)^2 - (\sqrt{2})^2 = 0\)

\(x^2 - 2x + 1 - 2 = 0\)

\(x^2 - 2x - 1 = 0\)

Any non-zero multiple of this equation would also have these solutions.

146. \(x = -3 + \sqrt{5}, x = -3 - \sqrt{5}\), so:

\([(x - (-3 + \sqrt{5}))(x - (-3 - \sqrt{5})) = 0\)

\((x + 3 - \sqrt{5})(x + 3 + \sqrt{5}) = 0\)

\(x^2 + 6x + 4 = 0\)

147. \((10c)y = 10(xy)\) by the Associative Property of Multiplication.

148. \(-4(x - 3) = -4(x + 12)\) by the Distributive Property.
149. \(7x^4 + (-7x^4) = 0\) by the Additive Inverse Property.

150. \((x + 4) + x^3 = x + (4 + x^3)\) by the Associative Property of Addition.

151. \((x + 3)(x - 6) = x^2 - 6x + 3x - 18\)
\[= x^2 - 3x - 18\]

152. \((x - 8)(x - 1) = x^2 - x - 8x + (-8)(-1)\)
\[= x^2 - 9x + 8\]

153. \((x + 4)(x^2 - x + 2) = x(x^2 - x + 2) + 4(x^2 - x + 2)\)
\[= x^3 - x^2 + 2x + 4x^2 - 4x + 8\]
\[= x^3 + 3x^2 - 2x + 8\]

154. \((x + 9)(x^2 - 6x + 4) = x^3 - 6x^2 + 4x + 9x^2 - 54x + 36\)
\[= x^3 + 3x^2 - 50x + 36\]

155. \(x^5 - 27x^2 = x^2(x^3 - 27)\)
\[= x^2(x^3 - 3^3)\]
\[= x^2(x - 3)(x^2 + 3x + 9)\]

156. \(x^3 - 5x^2 - 14x = x(x^2 - 5x - 14)\)
\[= x(x - 7)(x + 2)\]

157. \(x^3 + 5x^2 - 2x - 10 = x^2(x + 5) - 2(x + 5)\)
\[= (x + 5)(x^2 - 2)\]

158. \(2x^3 + x^2 - 8x - 4 = (2x^3 + x^2) - (8x + 4)\)
\[= x^2(2x + 1) - 4(2x + 1)\]
\[= (x^2 - 4)(2x + 1)\]
\[= (x + 2)(x - 2)(2x + 1)\]

159. Answers will vary.

Section 1.5  Complex Numbers

- Standard form: \(a + bi\).
  If \(b = 0\), then \(a + bi\) is a real number.
  If \(a = 0\) and \(b \neq 0\), then \(a + bi\) is a pure imaginary number.
- Equality of Complex Numbers: \(a + bi = c + di\) if and only if \(a = c\) and \(b = d\)
- Operations on complex numbers
  (a) Addition: \((a + bi) + (c + di) = (a + c) + (b + d)i\)
  (b) Subtraction: \((a + bi) - (c + di) = (a - c) + (b - d)i\)
  (c) Multiplication: \((a + bi)(c + di) = (ac - bd) + (ad + bc)i\)
  (d) Division: \(\frac{a + bi}{c + di} = \frac{a + bi}{c + di} \cdot \frac{c - di}{c + di} = \frac{ac + bd + bc - ad}{c^2 + d^2}\)
- The complex conjugate of \(a + bi\) is \(a - bi\):
  \((a + bi)(a - bi) = a^2 + b^2\)
- The additive inverse of \(a + bi\) is \(-a - bi\).
- \(\sqrt{-a} = \sqrt{a}i\) for \(a > 0\).

Vocabulary Check
1. (a) iii  (b) i  (c) ii
2. \(\sqrt{-1}; -1\)
3. principal square
4. complex conjugates
1. $a + bi = -10 + 6i$
   \[ a = -10 \]
   \[ b = 6 \]

2. $a + bi = 13 + 4i$
   \[ a = 13 \]
   \[ b = 4 \]

3. $(a - 1) + (b + 3)i = 5 + 8i$
   \[ a - 1 = 5 \Rightarrow a = 6 \]
   \[ b + 3 = 8 \Rightarrow b = 5 \]

4. $(a + 6) + 2bi = 6 - 5i$
   \[ 2b = -5 \]
   \[ b = -\frac{5}{2} \]
   \[ a + 6 = 6 \]
   \[ a = 0 \]

5. $4 + \sqrt{-9} = 4 + 3i$

6. $3 + \sqrt{-16} = 3 + 4i$

7. $2 - \sqrt{-27} = 2 - \sqrt{27}i$
   \[ = 2 - 3\sqrt{3}i \]

8. $1 + \sqrt{-8} = 1 + 2\sqrt{2}i$

9. $\sqrt{-75} = \sqrt{15}i = 5\sqrt{3}i$

10. $\sqrt{-4} = 2i$

11. $8 = 8 + 0i = 8$

12. $45$

13. $-6i + i^2 = -6i - 1$
   \[ = -1 - 6i \]

14. $-4i^2 + 2i = -4(-1) + 2i$
   \[ = 4 + 2i \]

15. $\sqrt{-0.09} = \sqrt{0.09}i$
   \[ = 0.3i \]

16. $\sqrt{-0.0004} = 0.02i$

17. $(5 + i) + (6 - 2i) = 11 - i$

18. $(13 - 2i) + (-5 + 6i) = 8 + 4i$

19. $(8 - i) - (4 - i) = 8 - i - 4 + i$
   \[ = 4 \]

20. $(3 + 2i) - (6 + 13i) = 3 + 2i - 6 - 13i$
   \[ = -3 - 11i \]

21. $(-2 + \sqrt{-8}) + (5 - \sqrt{-50}) = -2 + 2\sqrt{2}i + 5 - 5\sqrt{2}i$
   \[ = 3 - 3\sqrt{2}i \]

22. $(8 + \sqrt{-18}) - (4 + 3\sqrt{2}i) = 8 + 3\sqrt{2}i - 4 - 3\sqrt{2}i$
   \[ = 4 \]

23. $13i - (14 - 7i) = 13i - 14 + 7i$
   \[ = -14 + 20i \]

24. $22 + (-5 + 8i) + 10i = 17 + 18i$

25. $-\left(\frac{3}{2} + \frac{5}{2}i\right) + (\frac{3}{2} + \frac{4}{2}i) = -\frac{3}{2} - \frac{5}{2}i + \frac{3}{2} + \frac{4}{2}i$
   \[ = \frac{3}{2} - \frac{15}{6}i + \frac{10}{6}i + \frac{22}{6}i \]
   \[ = \frac{3}{2} + \frac{7}{6}i \]

26. $(1.6 + 3.2i) + (-5.8 + 4.3i) = -4.2 + 7.5i$

27. $(1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2$
   \[ = 3 + i + 2 = 5 + i \]

28. $(6 - 2i)(2 - 3i) = 12 - 18i - 4i + 6i^2$
   \[ = 12 - 22i - 6 = 6 - 22i \]

29. $6i(5 - 2i) = 30i - 12i^2 = 30i + 12$
   \[ = 12 + 30i \]

30. $-8i(9 + 4i) = -72i - 32i^2$
   \[ = 32 - 72i \]

31. $(\sqrt{14} + \sqrt{10}i)(\sqrt{14} - \sqrt{10}i) = 14 - 10i^2$
   \[ = 14 + 10 = 24 \]
32. \((\sqrt{3} + \sqrt{5}i)(\sqrt{3} - \sqrt{5}i) = 3 - 15i^2\)
\[= 3 - 15(-1)\]
\[= 3 + 15 = 18\]

34. \((2 - 3i)^2 = 4 - 12i + 9i^2\)
\[= 4 - 9 - 12i\]
\[= -5 - 12i\]

36. \((1 - 2i)^2 - (1 + 2i)^2 = 1 - 4i + 4i^2 - (1 + 4i + 4i^2)\)
\[= 1 - 4i + 4i^2 - 1 - 4i - 4i^2\]
\[= -8i\]

38. The complex conjugate of \(7 - 12i\) is \(7 + 12i\).
\((7 - 12i)(7 + 12i) = 49 - 144i^2\)
\[= 49 \cdot (-144)\]
\[= 193\]

40. The complex conjugate of \(-3 + \sqrt{2}i\) is \(-3 - \sqrt{2}i\).
\((-3 + \sqrt{2}i)(-3 - \sqrt{2}i) = 9 - 2i^2\)
\[= 9 - (-2)\]
\[= 11\]

42. The complex conjugate of \(-\sqrt{15} = \sqrt{15}i\) is \(-\sqrt{15}i\).
\((-\sqrt{15})i(-\sqrt{15}i) = -15i^2 = -(-15) = 15\]

44. The complex conjugate of \(1 + \sqrt{8}\) is \(1 + \sqrt{8}i\).
\((1 + \sqrt{8})(1 + \sqrt{8}i) = 1 + 2\sqrt{8}i + 8\)
\[= 9 + 4\sqrt{2}\]

46. \(-\frac{14}{2i} - \frac{-2i}{-2i} = \frac{28i}{-4i^2} = \frac{28i}{4} = 7i\]

48. \(\frac{5}{1 - i} \cdot \frac{1 + i}{1 + i} = \frac{5 + 5i}{1 - i^2} = \frac{5 + 5i}{2} = \frac{5}{2} + \frac{5}{2}i\]

50. \(\frac{6 - 7i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} = \frac{6 + 12i - 7i - 14i^2}{1 - 4i^2}\)
\[= \frac{20 + 5i}{5} = \frac{20}{5} + \frac{5}{5}i = 4 + i\]

33. \((4 + 5i)^2 = 16 + 40i + 25i^2\)
\[= 16 + 40i - 25\]
\[= -9 + 40i\]

35. \((2 + 3i)^2 + (2 - 3i)^2 = 4 + 12i + 9i^2 + 4 - 12i + 9i^2\)
\[= 4 + 12i - 9 + 4 - 12i - 9\]
\[= -10\]

37. The complex conjugate of \(6 + 3i\) is \(6 - 3i\).
\((6 + 3i)(6 - 3i) = 36 - (3i)^2 = 36 + 9 = 45\)

39. The complex conjugate of \(-1 - \sqrt{5}i\) is \(-1 + \sqrt{5}i\).
\((-1 - \sqrt{5}i)(-1 + \sqrt{5}i) = (1)^2 - (\sqrt{5}i)^2\)
\[= 1 + 5 = 6\]

41. The complex conjugate of \(\sqrt{20} = 2\sqrt{5}i\) is \(-2\sqrt{5}i\).
\((2\sqrt{5}i)(-2\sqrt{5}i) = -20i^2 = 20\)

43. The complex conjugate of \(\sqrt{8}\) is \(-\sqrt{8}\).
\((\sqrt{8})(-\sqrt{8}) = 8\)

45. \(\frac{5}{i} = \frac{5}{i} \cdot \frac{-i}{-i} = \frac{-5i}{1} = -5i\)

47. \(\frac{2}{4 - 5i} = \frac{2}{4 - 5i} \cdot \frac{4 + 5i}{4 + 5i}\)
\[= \frac{2(4 + 5i)}{16 + 25} = \frac{8 + 10i}{41} = \frac{8}{41} + \frac{10}{41}i\]

49. \(\frac{3 + i}{3 - i} = \frac{3 + i}{3 - i} \cdot \frac{3 + i}{3 + i}\)
\[= \frac{9 + 6i + i^2}{9 + 1} = \frac{8 + 6i}{10} = \frac{4}{5} + \frac{3}{5i}\]

51. \(\frac{6 - 5i}{i} = \frac{6 - 5i}{i} \cdot \frac{-i}{-i}\)
\[= \frac{-6i + 5i^2}{1} = -5 - 6i\]
52. \( \frac{8 + 16i}{2i} \cdot \frac{-2i}{-2i} = \frac{-16i - 32i^2}{-4i^2} = 8 - 4i \)

53. \( \frac{3i}{(4 - 5i)^2} = \frac{3i}{16 - 40i + 25i^2} = \frac{3i}{-9 - 40i} \cdot \frac{-9 + 40i}{-9 + 40i} \)
\[
\begin{align*}
&= \frac{-27i + 120i^2}{81 + 1600} = \frac{-120 - 27i}{1681} \\
&= \frac{-120}{1681} \cdot \frac{-27i}{1681} \\
&= \frac{-5}{1} \cdot \frac{-49}{1681}i \\
&= \frac{5}{1681} \cdot \frac{-49}{1681}i \\
&= \frac{5}{1681} \cdot \frac{-49}{1681}i
\end{align*}
\]

54. \( \frac{5i}{(2 + 3i)^2} = \frac{5i}{4 + 12i + 9i^2} \)
\[
\begin{align*}
&= \frac{5i}{-5 + 12i} \cdot \frac{-5 - 12i}{-5 - 12i} \\
&= \frac{-25i - 60i^2}{25 - 144i^2} \\
&= \frac{60 - 25i}{60} \cdot \frac{60}{25}i \\
&= \frac{5}{16} \cdot \frac{60}{25}i \\
&= \frac{5}{16} \cdot \frac{60}{25}i
\end{align*}
\]

55. \( \frac{2}{1 + i} - \frac{3}{1 - i} = \frac{2(1 - i) - 3(1 + i)}{(1 + i)(1 - i)} \)
\[
\begin{align*}
&= \frac{2 - 2i - 3 - 3i}{1 + 1} \\
&= \frac{-1 - 5i}{2} \\
&= \frac{-1}{2} - \frac{5}{2}i
\end{align*}
\]

56. \( \frac{2i}{2 + i} + \frac{5}{2 - i} = \frac{2i(2 - i)}{(2 + i)(2 - i)} + \frac{5(2 + i)}{(2 + i)(2 - i)} \)
\[
\begin{align*}
&= \frac{4i - 2i^2 + 10 + 5i}{4 - i^2} \\
&= \frac{12 + 9i}{5} \\
&= \frac{12}{5} + \frac{9}{5}i
\end{align*}
\]

57. \( \frac{i}{3 - 2i} + \frac{2i}{3 + 8i} = \frac{i(3 + 8i) + 2i(3 - 2i)}{(3 - 2i)(3 + 8i)} \)
\[
\begin{align*}
&= \frac{3i + 8i^2 + 6i - 4i^2}{9 + 24i - 6i - 16i^2} \\
&= \frac{4i^2 + 9i}{9 + 18i + 16} \\
&= \frac{-4 + 9i}{25 - 18i} \cdot \frac{25 + 18i}{25 - 18i} \\
&= \frac{-100 + 72i + 225i - 162i^2}{625 + 324} \\
&= \frac{-100 + 297i + 162}{949} \\
&= \frac{62 + 329i}{949} = \frac{62}{949} + \frac{329}{949}i
\end{align*}
\]

58. \( \frac{1 + i}{i} - \frac{3}{4 - i} = \frac{(1 + i)(4 - i) - 3i}{i(4 - i)} \)
\[
\begin{align*}
&= \frac{4 - i + 4i - i^2 - 3i}{4i - i^2} \\
&= \frac{4 - i + 4i - i^2}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i} \\
&= \frac{5}{1} \cdot \frac{1 - 4i}{1 - 4i} \\
&= \frac{5}{1} \cdot \frac{1 - 4i}{1 - 4i} \\
&= \frac{5}{17} \cdot \frac{20}{17} \\
&= \frac{5}{17} + \frac{20}{17}
\end{align*}
\]

59. \( \sqrt{-6} \cdot \sqrt{-2} = (\sqrt{6i})(\sqrt{2i}) = \sqrt{12i^2} = (2\sqrt{3})(-1) \)
\[
= -2\sqrt{3}
\]

60. \( \sqrt{-3} \cdot \sqrt{-10} = (\sqrt{3i})(\sqrt{-10i}) \)
\[
= \sqrt{30i^2} = 5\sqrt{2}(-1) = -5\sqrt{2}
\]

61. \( (\sqrt{-10})^2 = (\sqrt{10i})^2 = 10i^2 = -10 \)

62. \( (\sqrt{-75})^2 = (\sqrt{75i})^2 = 75i^2 = -75 \)

63. \( (3 + \sqrt{3})(7 - \sqrt{-10}) = (3 + \sqrt{3})(7 - \sqrt{10i}) \)
\[
= 21 - 3\sqrt{10i} + 7\sqrt{3} - \sqrt{30i^2} \\
= (21 + 3\sqrt{3}) + (7\sqrt{3} - 3\sqrt{10})i \\
= (21 + 5\sqrt{2}) + (7\sqrt{3} - 3\sqrt{10})i
\]
64. \((2 - \sqrt{-6})^2 = (2 - \sqrt{6i})(2 - \sqrt{6i})\)
   \[= 4 - 2\sqrt{6i} - 2\sqrt{6i} + 6i^2\]
   \[= 4 - 2\sqrt{6i} - 2\sqrt{6i} + 6(-1)\]
   \[= 4 - 6 - 4\sqrt{6i}\]
   \[= -2 - 4\sqrt{6i}\]

65. \(x^2 - 2x + 2 = 0; \ a = 1, \ b = -2, \ c = 2\)
   \[x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}\]
   \[= \frac{2 \pm \sqrt{-4}}{2}\]
   \[= \frac{2 \pm 2i}{2}\]
   \[= 1 \pm i\]

66. \(x^2 + 6x + 10 = 0; \ a = 1, \ b = 6, \ c = 10\)
   \[x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}\]
   \[= \frac{-6 \pm \sqrt{-4}}{2}\]
   \[= -3 \pm i\]

67. \(4x^2 + 16x + 17 = 0; \ a = 4, \ b = 16, \ c = 17\)
   \[x = \frac{-16 \pm \sqrt{(-16)^2 - 4(4)(17)}}{2(4)}\]
   \[= \frac{-16 \pm \sqrt{-16}}{8}\]
   \[= \frac{-16 \pm 4i}{8} = -2 \pm \frac{1}{2}i\]

68. \(9x^2 - 6x + 37 = 0; \ a = 9, \ b = -6, \ c = 37\)
   \[x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)}\]
   \[= \frac{6 \pm \sqrt{-1296}}{18}\]
   \[= \frac{1}{3} \pm \frac{36i}{18} = \frac{1}{3} \pm 2i\]

69. \(4x^2 + 16x + 15 = 0; \ a = 4, \ b = 16, \ c = 15\)
   \[x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)}\]
   \[= \frac{-16 \pm \sqrt{16}}{8}\]
   \[= \frac{-16 \pm 4}{8}\]
   \[= \frac{-12}{8} = \frac{3}{2} \quad \text{or} \quad x = \frac{20}{8} = \frac{5}{2}\]

70. \(16t^2 - 4t + 3 = 0; \ a = 16, \ b = -4, \ c = 3\)
   \[t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}\]
   \[= \frac{4 \pm \sqrt{-176}}{32}\]
   \[= \frac{4 \pm 4\sqrt{11}i}{32}\]
   \[= \frac{1}{8} \pm \frac{\sqrt{11}i}{8}\]

71. \(\frac{3}{2}x^2 - 6x + 9 = 0\)  
   Multiply both sides by 2.
   \(3x^2 - 12x + 18 = 0\)
   \[x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}\]
   \[= \frac{12 \pm \sqrt{-72}}{6}\]
   \[= \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i\]

72. \(\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0\)
   \[14x^2 - 12x + 5 = 0; \ a = 14, \ b = -12, \ c = 5\]
   \[x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)}\]
   \[= \frac{12 \pm \sqrt{136}}{28}\]
   \[= \frac{12 \pm 2\sqrt{34}}{28}\]
   \[= \frac{3}{7} \pm \frac{\sqrt{34}}{14}i\]

73. \(1.4x^2 - 2x - 10 = 0\)  
   Multiply both sides by 5.
   \(7x^2 - 10x - 50 = 0\)
   \[x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)}\]
   \[= \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14}\]
   \[= \frac{5 \pm 5\sqrt{15}}{7} = \frac{5}{7} \pm \frac{5\sqrt{15}}{7}i\]
74. \( 4.5x^2 - 3x + 12 = 0; \ a = 4.5, b = -3, c = 12 \)

\[
x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)}
\]

\[
= \frac{3 \pm \sqrt{-207}}{9} = \frac{3 \pm 3i\sqrt{23}}{9} = \frac{1}{3} \pm \frac{\sqrt{23}}{3}i
\]

75. \(-6i^3 + i^2 = -6i^2 + i^2 = -6(-1)i + (-1) = 6i - 1 = -1 + 6i\)

76. \(4i^2 - 2i^3 = -4 + 2i\)

77. \(-5i^5 = -5i^4i = -5(-1)i = 5i\)

78. \((-i)^3 = (-1)(i^2) = (-1)(-i) = i\)

79. \((\sqrt{-75})^3 = (5\sqrt{3}i)^3 = 5(\sqrt{3}i)^3 = 125(\sqrt{3}i)(-1)i = -375\sqrt{3}i\)

80. \((\sqrt{-2})^6 = (\sqrt{2}i)^6 = 8i^6 = 8i^4i^2 = -8\)

81. \(\frac{1}{i^5} = \frac{1}{i} = \frac{i}{-i} = i = -i = \frac{i}{1} = i\)

82. \(\frac{1}{2i} = \frac{1}{8i^3} = \frac{1}{8i} \cdot \frac{8i}{8i} = \frac{8i}{-64i^2} = \frac{8i}{64} = \frac{1}{8}i\)

83. (a) \(z_1 = 9 + 16i, z_2 = 20 - 10i\)

(b) \[
\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{9 + 16i} + \frac{1}{20 - 10i} = \frac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \frac{29 + 6i}{340 + 230i}
\]

\[
z = \left(\frac{340 + 230i}{29 + 6i}\right) \left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11,240 + 4630i}{877} = \frac{11,240 + 4630}{877} = \frac{160}{1}
\]

84. (a) \(2^3 = 8\)

(b) \((-1 + \sqrt{3}i)^3 = (-1)^3 + 3(-1)^2(\sqrt{3}i) + 3(-1)(\sqrt{3}i)^2 + (\sqrt{3}i)^3 = -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3 = -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i = 8\)

(c) \((-1 - \sqrt{3}i)^3 = (-1)^3 + 3(-1)^2(-\sqrt{3}i) + 3(-1)(-\sqrt{3}i)^2 + (-\sqrt{3}i)^3 = -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3 = -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i = 8\)

85. (a) \(2^4 = 16\)

(b) \((-2)^4 = 16\)

(c) \((2i)^4 = 2^4i^4 = 16(1) = 16\)

(d) \((-2i)^4 = (-2)^4i^4 = 16(1) = 16\)

86. (a) \(i^{40} = (i^4)^{10} = (1)^{10} = 1\)

(b) \(i^{25} = (i^4)^{6} \cdot i = (1)^{6}i = i\)

(c) \(i^{30} = (i^4)^{12}(i^2) = (1)(-1) = -1\)

(d) \(i^{87} = (i^4)^{21}(i^3) = (1)(i^3) = -i\)
87. False, if \( b = 0 \) then \( a + bi = a - bi = a \).

That is, if the complex number is real, the number equals its conjugate.

88. True

\[
x^4 - x^2 + 14 = 56
\]

\[
\left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 = 56
\]

\[
36 + 6 + 14 = 56
\]

56 = 56

89. False

\[
4^{14} + 4^{20} - 4^{74} - 4^{109} + 4^{1} = (4^{44})^{11} + (4^{5})^{37}(4) - (4^{5})^{37}(4) - (4^{44})^{15}(4)
\]

\[
= (1)^{11} + (1)^{37}(-1) - (1)^{37}(-1) - (1)^{15}(1)
\]

\[
= 1 + (-1) + 1 - 1 + i = 1
\]

90. \( \sqrt{6} \sqrt{6} = \sqrt{6} \sqrt{6} = 6i^2 = -6 \)

91. \((a_1 + b_1)(a_2 + b_2) = a_1a_2 + a_1b_2 + a_2b_1 + b_1b_2)\)

\[
= (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i
\]

The complex conjugate of this product is \((a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i\).

The product of the complex conjugates is:

\[
(a_1 - b_1)(a_2 - b_2) = a_1a_2 - a_1b_2 - a_2b_1 + b_1b_2i
\]

\[
= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i
\]

Thus, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

92. \((a_1 + b_1)(a_2 + b_2) = (a_1 + a_2) + (b_1 + b_2)i\)

The complex conjugate of this sum is \((a_1 + a_2) - (b_1 + b_2)i\).

The sum of the complex conjugates is \((a_1 - b_1)(a_2 - b_2) = (a_1 + a_2) - (b_1 + b_2)i\).

Thus, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

93. \((4 + 3x) + (8 - 6x - x^2) = -x^2 - 3x + 12\)

94. \((x^3 - 3x^2) - (6 - 2x - 4x^2) = x^3 - 3x^2 - 6 + 2x + 4x^2\)

\[
x^3 + x^2 + 2x - 6
\]

95. \((3x - \frac{1}{2})(x + 4) = 3x^2 + 12x - \frac{1}{2}x - 2\)

\[
x^3 + 2x^2 - 12x - 2
\]

96. \((2x - 5)^2 = (2x)^2 - 2(2x)(5) + (5)^2\)

\[
= 4x^2 - 20x + 25
\]

97. \(-x - 12 = 19\)

\[
x = 31
\]

98. \(8 - 3x = -34\)

\[
x = 14
\]

99. \(4(5x - 6) - 3(6x + 1) = 0\)

\[
20x - 24 - 18x - 3 = 0
\]

\[
2x - 27 = 0
\]

\[
x = 27
\]

100. \(5(x - (3x + 11)) = 20x - 15\)

\[
5x - 15x - 55 = 20x - 15
\]

\[
-30x = 40
\]

\[
x = 40
\]

\[
-30 = -\frac{4}{3}
\]

101. \(V = \frac{4}{3}\pi a^2 b\)

\[
\frac{3V}{4\pi b} = a^2
\]

\[
\sqrt{\frac{3V}{4\pi b}} = a
\]

\[
a = \frac{1}{2} \sqrt{\frac{3V}{\pi b}} = \frac{\sqrt{3V\pi b}}{2\pi b}
\]
102. \( F = \frac{am_1m_2}{r^2} \)

\[ r^2 = \frac{am_1m_2}{F} \]

\[ r = \sqrt{\frac{am_1m_2}{F}} = \sqrt[2]{am_1m_2} \cdot \sqrt[2]{F} = \frac{\sqrt{am_1m_2}}{F} \]

103. Let \( x \) liters withdrawn and replaced.

\[ 0.50(5 - x) + 1.00x = 0.60(5) \]
\[ 2.50 - 0.50x + 1.00x = 3.00 \]
\[ 0.50x = 0.50 \]
\[ x = 1 \text{ liter} \]

Section 1.6 Other Types of Equations

- You should be able to solve certain types of nonlinear or nonquadratic equations by rewriting them in a form in which you can factor, extract square roots, complete the square, or use the Quadratic Formula.
- For equations involving radicals or rational exponents, raise both sides to the same power.
- For equations that are of the quadratic type, \( au^2 + bu + c = 0, \ a \neq 0 \), use either factoring, the Quadratic Formula, or completing the square.
- For equations with fractions, multiply both sides by the least common denominator to clear the fractions.
- For equations involving absolute value, remember that the expression inside the absolute value can be positive or negative.
- Always check for extraneous solutions.

Vocabulary Check

1. polynomial
2. extraneous
3. quadratic type

1. \( 4x^4 - 18x^2 = 0 \)
\[ 2x^2(2x^2 - 9) = 0 \]
\[ 2x^2 = 0 \implies x = 0 \]
\[ 2x^2 - 9 = 0 \implies x = \pm \frac{3\sqrt{3}}{2} \]

2. \( 20x^3 - 125x = 0 \)
\[ 5x(4x^2 - 25) = 0 \]
\[ 5x = 0 \implies x = 0 \]
\[ 2x + 5 = 0 \implies x = -\frac{5}{2} \]
\[ 2x - 5 = 0 \implies x = \frac{5}{2} \]

3. \( x^4 - 81 = 0 \)
\[ (x^2 + 9)(x + 3)(x - 3) = 0 \]
\[ x^2 + 9 = 0 \implies x = \pm 3i \]
\[ x + 3 = 0 \implies x = -3 \]
\[ x - 3 = 0 \implies x = 3 \]

4. \( x^6 - 64 = 0 \)
\[ (x^3 - 8)(x^3 + 8) = 0 \]
\[ (x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4) = 0 \]
\[ x - 2 = 0 \implies x = 2 \]
\[ x^2 + 2x + 4 = 0 \implies x = -1 \pm \sqrt{3}i \]
\[ x + 2 = 0 \implies x = -2 \]
\[ x^2 - 2x + 4 = 0 \implies x = 1 \pm \sqrt{3}i \]
5. \[x^3 + 216 = 0\]
\[x^3 + 6^3 = 0\]
\[(x + 6)(x^2 - 6x + 36) = 0\]
\[x + 6 = 0 \Rightarrow x = -6\]
\[x^2 - 6x + 36 = 0 \Rightarrow x = 3 \pm 3i\]

6. \[27x^3 - 512 = 0\]
\[(3x - 8)(9x^2 + 24x + 64) = 0\]
\[3x - 8 = 0 \Rightarrow x = \frac{8}{3}\]
\[9x^2 + 24x + 64 = 0 \Rightarrow x = \frac{-24 \pm \sqrt{(24)^2 - 4(9)(64)}}{2(9)}\]
\[= \frac{-24 \pm \sqrt{-1728}}{18}\]
\[= \frac{-24 \pm 24\sqrt{3}i}{18}\]
\[= \frac{-4}{3} \pm 4\sqrt{3}i\]

7. \[5x^3 + 30x^2 + 45x = 0\]
\[5x(x^2 + 6x + 9) = 0\]
\[5x(x + 3)^2 = 0\]
\[5x = 0 \Rightarrow x = 0\]
\[x + 3 = 0 \Rightarrow x = -3\]

8. \[9x^4 - 24x^3 + 16x^2 = 0\]
\[x^2(9x^2 - 24x + 16) = 0\]
\[x^2(3x - 4)^2 = 0\]
\[x^2 = 0 \Rightarrow x = 0\]
\[3x - 4 = 0 \Rightarrow x = \frac{4}{3}\]

9. \[x^3 - 3x^2 - x + 3 = 0\]
\[x^2(x - 3) - (x - 3) = 0\]
\[(x - 3)(x^2 - 1) = 0\]
\[(x - 3)(x + 1)(x - 1) = 0\]
\[x - 3 = 0 \Rightarrow x = 3\]
\[x + 1 = 0 \Rightarrow x = -1\]
\[x - 1 = 0 \Rightarrow x = 1\]

10. \[x^3 + 2x^2 + 3x + 6 = 0\]
\[x^2(x + 2) + 3(x + 2) = 0\]
\[(x + 2)(x^2 + 3) = 0\]
\[x + 2 = 0 \Rightarrow x = -2\]
\[x^2 + 3 = 0 \Rightarrow x = \pm \sqrt{3}i\]

11. \[x^4 - x^3 + x - 1 = 0\]
\[x^3(x - 1) + (x - 1) = 0\]
\[(x - 1)(x^3 + 1) = 0\]
\[(x - 1)(x + 1)(x^2 - x + 1) = 0\]
\[x - 1 = 0 \Rightarrow x = 1\]
\[x + 1 = 0 \Rightarrow x = -1\]
\[x^2 - x + 1 = 0 \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i\]
12. \[ x^4 + 2x^3 - 8x - 16 = 0 \]
\[ x^3(x + 2) - 8(x + 2) = 0 \]
\[ (x^3 - 8)(x + 2) = 0 \]
\[ (x - 2)(x^2 + 2x + 4)(x + 2) = 0 \]
\[ x - 2 = 0 \Rightarrow x = 2 \]
\[ x^2 + 2x + 4 = 0 \Rightarrow x = -1 \pm \sqrt{3}i \]
\[ x + 2 = 0 \Rightarrow x = -2 \]

13. \[ x^4 - 4x^2 + 3 = 0 \]
\[ (x^2 - 3)(x^2 - 1) = 0 \]
\[ (x + \sqrt{3})(x - \sqrt{3})(x + 1)(x - 1) = 0 \]
\[ x + \sqrt{3} = 0 \Rightarrow x = -\sqrt{3} \]
\[ x - \sqrt{3} = 0 \Rightarrow x = \sqrt{3} \]
\[ x + 1 = 0 \Rightarrow x = -1 \]
\[ x - 1 = 0 \Rightarrow x = 1 \]

14. \[ x^4 + 5x^2 - 36 = 0 \]
\[ (x^2 + 9)(x^2 - 4) = 0 \]
\[ (x^2 + 9)(x + 2)(x - 2) = 0 \]
\[ x^2 + 9 = 0 \Rightarrow x = \pm 3i \]
\[ x + 2 = 0 \Rightarrow x = -2 \]
\[ x - 2 = 0 \Rightarrow x = 2 \]

15. \[ 4x^4 - 65x^2 + 16 = 0 \]
\[ (4x^2 - 1)(x^2 - 16) = 0 \]
\[ (2x + 1)(2x - 1)(x + 4)(x - 4) = 0 \]
\[ 2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \]
\[ 2x - 1 = 0 \Rightarrow x = \frac{1}{2} \]
\[ x + 4 = 0 \Rightarrow x = -4 \]
\[ x - 4 = 0 \Rightarrow x = 4 \]

16. \[ 36t^4 + 29t^2 - 7 = 0 \]
\[ (36t^2 - 7)(t^2 + 1) = 0 \]
\[ (6t + \sqrt{7})(6t - \sqrt{7})(t^2 + 1) = 0 \]
\[ 6t + \sqrt{7} = 0 \Rightarrow t = -\frac{\sqrt{7}}{6} \]
\[ 6t - \sqrt{7} = 0 \Rightarrow t = \frac{\sqrt{7}}{6} \]
\[ t^2 + 1 = 0 \Rightarrow t = \pm i \]

17. \[ x^6 + 7x^3 - 8 = 0 \]
\[ (x^3 + 8)(x^3 - 1) = 0 \]
\[ (x + 2)(x^2 - 2x + 4)(x - 1)(x^2 + x + 1) = 0 \]
\[ x + 2 = 0 \Rightarrow x = -2 \]
\[ x^2 - 2x + 4 = 0 \Rightarrow x = 1 \pm \sqrt{3}i \]
\[ x - 1 = 0 \Rightarrow x = 1 \]
\[ x^2 + x + 1 = 0 \Rightarrow x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i \]

18. \[ x^6 + 3x^3 + 2 = 0 \]
\[ (x^3 + 2)(x^3 + 1) = 0 \]
\[ (x + \sqrt{2})(x^2 - \sqrt{2}x + (\sqrt{2})^2)(x^3 + 1)(x^2 - x + 1) = 0 \]
\[ x + \sqrt{2} = 0 \Rightarrow x = -\sqrt{2} \]
\[ x^2 - \sqrt{2}x + (\sqrt{2})^2 = 0 \Rightarrow x = \frac{1 \pm \sqrt{3}i}{2^{3/2}} \]
\[ x + 1 = 0 \Rightarrow x = -1 \]
\[ x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{3}i}{2} \]
19. \[
\frac{1}{x^2} + \frac{8}{x} + 15 = 0 \\
1 + 8x + 15x^2 = 0 \\
(1 + 3x)(1 + 5x) = 0 \\
1 + 3x = 0 \implies x = -\frac{1}{3} \\
1 + 5x = 0 \implies x = -\frac{1}{5}
\]

20. \[
6\left(\frac{x}{x + 1}\right)^2 + 5\left(\frac{x}{x + 1}\right) - 6 = 0 \\
\text{Let } u = \frac{x}{x + 1}. \\
6u^2 + 5u - 6 = 0 \\
(3u - 2)(2u + 3) = 0 \\
3u - 2 = 0 \implies u = \frac{2}{3} \\
2u + 3 = 0 \implies u = -\frac{3}{2} \\
\frac{x}{x + 1} = \frac{2}{3} \implies x = 2 \\
\frac{x}{x + 1} = -\frac{3}{2} \implies x = -\frac{3}{5}
\]

21. \[
2x + 9\sqrt{x} = 5 \\
2x + 9\sqrt{x} - 5 = 0 \\
2\left(\sqrt{x}\right)^2 + 9\sqrt{x} - 5 = 0 \\
(2\sqrt{x} - 1)(\sqrt{x} + 5) = 0 \\
\sqrt{x} = \frac{1}{2} \implies x = \frac{1}{4} \\
(\sqrt{x} = -5 \text{ is not a solution.})
\]

22. \[
6x - 7\sqrt{x} - 3 = 0 \\
\text{Let } u = \sqrt{x}. \\
6u^2 - 7u - 3 = 0 \\
(3u + 1)(2u - 3) = 0 \\
3u + 1 = 0 \implies u = -\frac{1}{3} \\
2u - 3 = 0 \implies u = \frac{3}{2} \\
\sqrt{x} = -\frac{1}{3} \text{ is not a solution.} \\
\sqrt{x} = \frac{3}{2} \implies x = \frac{9}{4}
\]

23. \[
3x^{1/3} + 2x^{2/3} = 5 \\
2x^{2/3} + 3x^{1/3} - 5 = 0 \\
2(x^{1/3})^2 + 3x^{1/3} - 5 = 0 \\
(2x^{1/3} + 5)(x^{1/3} - 1) = 0 \\
2x^{1/3} + 5 = 0 \implies x^{1/3} = -\frac{5}{2} \implies x = \left(-\frac{5}{2}\right)^3 = -\frac{125}{8} \\
x^{1/3} - 1 = 0 \implies x^{1/3} = 1 \implies x = (1)^3 = 1
\]

24. \[
9t^{2/3} + 24t^{1/3} + 16 = 0 \\
(3t^{1/3} + 4)^2 = 0 \\
3t^{1/3} + 4 = 0 \implies t^{1/3} = -\frac{4}{3} \\
t = -\frac{64}{27}
\]

25. \[y = x^3 - 2x^2 - 3x \]

(a) ![Graph](image)

(b) \(x\)-intercepts: \((-1, 0), (0, 0), (3, 0)\)

(c) \(0 = x^3 - 2x^2 - 3x\) \\
\[0 = x(x + 1)(x - 3)\] \\
\[x = 0\] \\
x + 1 = 0 \implies x = -1 \\
x - 3 = 0 \implies x = 3

(d) The \(x\)-intercepts of the graph are the same as the solutions to the equation.
26. (a) 

(b) $x$-intercepts: $(0, 0), (\frac{3}{2}, 0), (6, 0)$

(c) $y = 2x^4 - 15x^3 + 18x^2$

$$0 = 2x^4 - 15x^3 + 18x^2$$

$$= x^2(2x^2 - 15x + 18)$$

$$= x^2(2x - 3)(x - 6)$$

$0 = x^2 \implies x = 0$

$0 = 2x - 3 \implies x = \frac{3}{2}$

$0 = x - 6 \implies x = 6$

$x = 0, \frac{3}{2}, 6$

(d) The $x$-intercepts and the solutions are the same.

27. $y = x^4 - 10x^2 + 9$

(a) 

(b) $x$-intercepts: $(\pm 1, 0), (\pm 3, 0)$

28. (a) 

(b) $x$-intercepts: $(-2, 0), (2, 0), (-5, 0), (5, 0)$

(d) The $x$-intercepts and the solutions are the same.

29. $\sqrt{5}x - 10 = 0$

$$\sqrt{5}x = 10$$

$$2x = 100$$

$$x = 50$$

30. $4\sqrt{x} - 3 = 0$

$$4\sqrt{x} = 3$$

$$16x = 9$$

$$x = \frac{9}{16}$$

31. $\sqrt{x - 10} - 4 = 0$

$$\sqrt{x - 10} = 4$$

$$x - 10 = 16$$

$$x = 26$$

32. $\sqrt{5} - x - 3 = 0$

$$\sqrt{5} - x = 3$$

$$5 - x = 9$$

$$x = -4$$

33. $\sqrt{2x + 5} + 3 = 0$

$$\sqrt{2x + 5} = -3$$

$$2x + 5 = -27$$

$$x = -16$$

34. $\sqrt{3x + 1} - 5 = 0$

$$\sqrt{3x + 1} = 5$$

$$3x + 1 = 25$$

$$x = \frac{124}{3}$$
35. \[ -\sqrt{26} - 11x + 4 = x \]
\[ 4 - x = \sqrt{26} - 11x \]
\[ 16 - 8x + x^2 = 26 - 11x \]
\[ x^2 + 3x - 10 = 0 \]
\[ (x + 5)(x - 2) = 0 \]
\[ x + 5 = 0 \implies x = -5 \]
\[ x - 2 = 0 \implies x = 2 \]

36. \[ x + \sqrt{31 - 9x} = 5 \]
\[ \sqrt{31 - 9x} = 5 - x \]
\[ 31 - 9x = 25 - 10x + x^2 \]
\[ 0 = x^2 - x - 6 \]
\[ 0 = (x - 3)(x + 2) \]
\[ 0 = x - 3 \implies x = 3 \]
\[ 0 = x - 2 \implies x = -2 \]

37. \[ \sqrt{x + 1} = \sqrt{3x + 1} \]
\[ x + 1 = 3x + 1 \]
\[ -2x = 0 \]
\[ x = 0 \]

38. \[ \sqrt{x + 5} = \sqrt{x - 5} \]
\[ x + 5 = x - 5 \]
\[ 5 = -5 \]
No solution

39. \[ \sqrt{x} - \sqrt{x - 5} = 1 \]
\[ \sqrt{x} = 1 + \sqrt{x - 5} \]
\[ (\sqrt{x})^2 = (1 + \sqrt{x - 5})^2 \]
\[ x = 1 + 2\sqrt{x - 5} + x - 5 \]
\[ 4 = 2\sqrt{x - 5} \]
\[ 2 = \sqrt{x - 5} \]
\[ 4 = x - 5 \]
\[ 9 = x \]

40. \[ \sqrt{x} + \sqrt{x - 20} = 10 \]
\[ \sqrt{x} = 10 - \sqrt{x - 20} \]
\[ (\sqrt{x})^2 = (10 - \sqrt{x - 20})^2 \]
\[ x = 100 - 20\sqrt{x - 20} + x - 20 \]
\[ -80 = -20\sqrt{x - 20} \]
\[ 4 = \sqrt{x - 20} \]
\[ 16 = x - 20 \]
\[ 36 = x \]

41. \[ \sqrt{x + 5} + \sqrt{x - 5} = 10 \]
\[ \sqrt{x + 5} = 10 - \sqrt{x - 5} \]
\[ (\sqrt{x + 5})^2 = (10 - \sqrt{x - 5})^2 \]
\[ x + 5 = 100 - 20\sqrt{x - 5} + x - 5 \]
\[ -90 = -20\sqrt{x - 5} \]
\[ 9 = 2\sqrt{x - 5} \]
\[ 81 = 4(x - 5) \]
\[ 81 = 4x - 20 \]
\[ 101 = 4x \]
\[ \frac{101}{4} = x \]

42. \[ 2\sqrt{x + 1} - \sqrt{2x + 3} = 1 \]
\[ 2\sqrt{x + 1} = 1 + \sqrt{2x + 3} \]
\[ (2\sqrt{x + 1})^2 = (1 + \sqrt{2x + 3})^2 \]
\[ 4(x + 1) = 1 + 2\sqrt{2x + 3} + 2x + 3 \]
\[ 2x = 2\sqrt{2x + 3} \]
\[ x = \sqrt{2x + 3} \]
\[ x^2 = 2x + 3 \]
\[ x^2 - 2x - 3 = 0 \]
\[ (x - 3)(x + 1) = 0 \]
\[ x - 3 = 0 \implies x = 3 \]
\[ x + 1 = 0 \implies x = -1, \text{ extraneous} \]
43. \( \sqrt{x + 2} - \sqrt{2x - 3} = -1 \)

\[
\begin{align*}
\sqrt{x + 2} &= \sqrt{2x - 3} - 1 \\
(\sqrt{x + 2})^2 &= (\sqrt{2x - 3} - 1)^2 \\
x + 2 &= 2x - 3 - 2\sqrt{2x - 3} + 1 \\
x + 2 &= 2x - 2 - 2\sqrt{2x - 3} \\
x - 4 &= -2\sqrt{2x - 3} \\
x - 4 &= 2\sqrt{2x - 3} \\
(x - 4)^2 &= (2\sqrt{2x - 3})^2 \\
x^2 - 8x + 16 &= 4(2x - 3) \\
x^2 - 8x + 16 &= 8x - 12 \\
x^2 - 16x + 28 &= 0 \\
(x - 2)(x - 14) &= 0 \\
x - 2 &= 0 \Rightarrow x = 2, \text{ extraneous} \\
x - 14 &= 0 \Rightarrow x = 14
\end{align*}
\]

44. \( 4\sqrt{x - 3} - \sqrt{6x - 17} = 3 \)

\[
\begin{align*}
4\sqrt{x - 3} &= 3 + \sqrt{6x - 17} \\
(4\sqrt{x - 3})^2 &= (3 + \sqrt{6x - 17})^2 \\
16(x - 3) &= 9 + 6\sqrt{6x - 17} + 6x - 17 \\
10x - 40 &= 6\sqrt{6x - 17} \\
(10x - 40)^2 &= (6\sqrt{6x - 17})^2 \\
100x^2 - 800x + 1600 &= 36(6x - 17) \\
100x^2 - 1016x + 2212 &= 0 \\
25x^2 - 254x + 553 &= 0 \\
(25x - 79)(x - 7) &= 0 \\
25x - 79 &= 0 \Rightarrow x = \frac{79}{25}, \text{ extraneous} \\
x - 7 &= 0 \Rightarrow x = 7
\end{align*}
\]

45. \( (x - 5)^{3/2} = 8 \)

\[
\begin{align*}
(x - 5)^3 &= 8^2 \\
x - 5 &= \sqrt[3]{64} \\
x &= 5 + 4 = 9
\end{align*}
\]

46. \( (x + 3)^{3/2} = 8 \)

\[
\begin{align*}
(x + 3)^3 &= 8^2 \\
x + 3 &= \sqrt[3]{64} \\
x &= -3 + 4 = 1
\end{align*}
\]

47. \( (x + 3)^{2/3} = 8 \)

\[
\begin{align*}
(x + 3)^2 &= 8^3 \\
(x + 3)^2 &= \sqrt[3]{8^3} \\
x + 3 &= \pm \sqrt[3]{512} \\
x &= -3 \pm 16\sqrt{2}
\end{align*}
\]

48. \( (x + 2)^{2/3} = 9 \)

\[
\begin{align*}
(x + 2)^2 &= 9^3 \\
x + 2 &= \pm \sqrt[3]{729} \\
x &= -2 \pm 27 = -29, 25
\end{align*}
\]

49. \( (x^2 - 5)^{1/2} = 27 \)

\[
\begin{align*}
(x^2 - 5)^2 &= 27^2 \\
x^2 - 5 &= \sqrt{27^2} \\
x^2 &= 5 + 9 \\
x &= \pm \sqrt{14}
\end{align*}
\]

50. \( (x^2 - x - 22)^{1/2} = 27 \)

\[
\begin{align*}
x^2 - x - 22 &= 27^{2/3} \\
x^2 - x - 22 &= 9 \\
x^2 - x - 31 &= 0 \\
x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-31)}}{2(1)} = \frac{1 \pm \sqrt{125}}{2} = \frac{1 \pm 5\sqrt{5}}{2}
\end{align*}
\]

51. \( 3\sqrt{x - 1} + 2(x - 1)^{1/2} = 0 \)

\[
\begin{align*}
(x - 1)^{1/2}[3x + 2(x - 1)] &= 0 \\
(x - 1)^{1/2}(5x - 2) &= 0 \\
(x - 1)^{1/2} &= 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1 \\
5x - 2 &= 0 \Rightarrow x = \frac{2}{5}, \text{ extraneous}
\end{align*}
\]

52. \( 4\sqrt{x - 1} + 6\sqrt{x - 1} = 0 \)

\[
\begin{align*}
2\sqrt{2(x - 1)} + 3(x - 1)^{1/2} &= 0 \\
2\sqrt{2(x - 1)} + 3(x - 1) &= 0 \\
2\sqrt{x - 1} &= 0 \Rightarrow x = 0 \\
x - 1 &= 0 \Rightarrow x = 1 \\
5x - 3 &= 0 \Rightarrow x = \frac{3}{5}
\end{align*}
\]
53. \( y = \sqrt{11x - 30} - x \)

(a) 

(b) \( x \)-intercepts: (5, 0), (6, 0)

(c) 

\[ 0 = \sqrt{11x - 30} - x \]
\[ x = \sqrt{11x - 30} \]
\[ x^2 = 11x - 30 \]
\[ x^2 - 11x + 30 = 0 \]
\[(x - 5)(x - 6) = 0 \]
\[ x - 5 = 0 \Rightarrow x = 5 \]
\[ x - 6 = 0 \Rightarrow x = 6 \]

(d) The \( x \)-intercepts of the graph are the same as the solutions to the equation.

55. \( y = \sqrt{\frac{7}{4}x + 36} - \sqrt{5x + 16} - 2 \)

(a) 

(b) \( x \)-intercepts: (0, 0), (4, 0)

(c) 

\[ 0 = \sqrt{\frac{7}{4}x + 36} - \sqrt{5x + 16} - 2 \]
\[ \frac{\sqrt{7}x + 36}{4} = -\sqrt{5x + 16} - 2 \]
\[ \sqrt{7x + 36} = 2 + \sqrt{5x + 16} \]
\[ (\sqrt{7x + 36})^2 = (2 + \sqrt{5x + 16})^2 \]
\[ 7x + 36 = 4 + 4\sqrt{5x + 16} + 5x + 16 \]
\[ 7x + 36 = 5x + 20 + 4\sqrt{5x + 16} \]
\[ 2x + 16 = 4\sqrt{5x + 16} \]
\[ x + 8 = 2\sqrt{5x + 16} \]
\[ (x + 8)^2 = (2\sqrt{5x + 16})^2 \]
\[ x^2 + 16x + 64 = 4(5x + 16) \]
\[ x^2 + 16x + 64 = 20x + 64 \]
\[ x^2 - 4x = 0 \]
\[ x(x - 4) = 0 \]
\[ x = 0 \]
\[ x - 4 = 0 \Rightarrow x = 4 \]

(d) The \( x \)-intercepts of the graph are the same as the solutions to the equation.

54. (a) 

(b) \( x \)-intercept: \( \left( \frac{3}{4}, 0 \right) \)

(c) \( y = 2x - \sqrt{15 - 4x} \)

\[ 0 = 2x - \sqrt{15 - 4x} \]
\[ \sqrt{15 - 4x} = 2x \]
\[ 15 - 4x = 4x^2 \]
\[ 0 = 4x^2 + 4x - 15 \]
\[ 0 = (2x + 5)(2x - 3) \]
\[ 0 = 2x + 5 \Rightarrow x = -\frac{5}{2}, \text{ extraneous} \]
\[ 0 = 2x - 3 \Rightarrow x = \frac{3}{2} \]
\[ x = \frac{3}{2} \]

(d) The \( x \)-intercept and the solution are the same.

56. (a) 

(b) \( x \)-intercept: (4, 0)

(c) \( y = 3\sqrt{x} - \frac{4}{\sqrt{x}} - 4 \)

\[ 0 = 3\sqrt{x} - \frac{4}{\sqrt{x}} - 4 \]
\[ 0 = \sqrt{x}\left(3\sqrt{x} - \frac{4}{\sqrt{x}} - 4\right) \]
\[ 0 = 3x - 4 - 4\sqrt{x} \]
\[ 0 = (3\sqrt{x} + 2)(\sqrt{x} - 2) \]
\[ 0 = 3\sqrt{x} + 2 \Rightarrow x = \frac{4}{9}, \text{ extraneous} \]
\[ 0 = \sqrt{x} - 2 \Rightarrow x = 4 \]
\[ x = 4 \]

(d) The \( x \)-intercept and the solution are the same.
57. \[ x = \frac{3}{x} + \frac{1}{2} \]

\[ (2x)(x) = (2x)\left(\frac{3}{x}\right) + (2x)\left(\frac{1}{2}\right) \]

\[ 2x^2 = 6 + x \]

\[ 2x^2 - x - 6 = 0 \]

\[ (2x + 3)(x - 2) = 0 \]

\[ 2x + 3 = 0 \implies x = -\frac{3}{2} \]

\[ x - 2 = 0 \implies x = 2 \]

59. \[ \frac{1}{x} - \frac{1}{x + 1} = 3 \]

\[ x(x + 1) - x(x + 1)\frac{1}{x + 1} = x(x + 1)(3) \]

\[ x + 1 - x = 3x(x + 1) \]

\[ 1 = 3x^2 + 3x \]

\[ 0 = 3x^2 + 3x - 1 \]

\[ a = 3, \ b = 3, \ c = -1 \]

\[ x = \frac{-3 \pm \sqrt{(3)^2 - 4(3)(-1)}}{2(3)} = \frac{-3 \pm \sqrt{21}}{6} \]

61. \[ \frac{20 - x}{x} = x \]

\[ 20 - x = x^2 \]

\[ 0 = x^2 + x - 20 \]

\[ 0 = (x + 5)(x - 4) \]

\[ x + 5 = 0 \implies x = -5 \]

\[ x - 4 = 0 \implies x = 4 \]

63. \[ \frac{x}{x^2 - 4} + \frac{1}{x + 2} = 3 \]

\[ (x + 2)(x - 2)\frac{x}{x^2 - 4} + (x + 2)(x - 2)\frac{1}{x + 2} = 3(x + 2)(x - 2) \]

\[ x + x - 2 = 3x^2 - 12 \]

\[ 3x^2 - 2x - 10 = 0 \]

\[ a = 3, \ b = -2, \ c = -10 \]

\[ x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-10)}}{2(3)} \]

\[ = \frac{2 \pm \sqrt{124}}{6} = \frac{2 \pm 2\sqrt{31}}{6} = \frac{1 \pm \sqrt{31}}{3} \]

62. \[ \frac{4x + 1}{x} = \frac{3}{x} \]

\[ (x)4x + (x)1 = (x)\frac{3}{x} \]

\[ 4x^2 + x = 3 \]

\[ 4x^2 + x - 3 = 0 \]

\[ (4x - 3)(x + 1) = 0 \]

\[ 4x - 3 = 0 \implies x = \frac{3}{4} \]

\[ x + 1 = 0 \implies x = -1 \]

64. \[ \frac{x + 1}{3} - \frac{x + 1}{x + 2} = 0 \]

\[ 3(x + 2)^2 + \frac{1}{3} - 3(x + 2)^2 + \frac{1}{x + 2} = 0 \]

\[ (x + 2)(x + 1) - 3(x + 1) = 0 \]

\[ x^2 + 3x + 2 - 3x - 3 = 0 \]

\[ x^2 - 1 = 0 \]

\[ (x + 1)(x - 1) = 0 \]

\[ x + 1 = 0 \implies x = -1 \]

\[ x - 1 = 0 \implies x = 1 \]
65. \[ |2x - 1| = 5 \]
   \[ 2x - 1 = 5 \Rightarrow x = 3 \]
   \[ -2x + 1 = 5 \Rightarrow x = -2 \]

66. \[ |3x + 2| = 7 \]
   \[ 3x + 2 = 7 \Rightarrow x = \frac{5}{3} \]
   \[ -3x - 2 = 7 \Rightarrow x = -3 \]

67. \[ |x| = x^2 + x - 3 \]

First equation:
   \[ x = x^2 + x - 3 \]
   \[ x^2 - 3 = 0 \]
   \[ x = \pm \sqrt{3} \]

Second equation:
   \[ -x = x^2 + x - 3 \]
   \[ x^2 + 2x - 3 = 0 \]
   \[ (x - 1)(x + 3) = 0 \]
   \[ x - 1 = 0 \Rightarrow x = 1 \]
   \[ x + 3 = 0 \Rightarrow x = -3 \]

Only \( x = \sqrt{3} \) and \( x = -3 \) are solutions to the original equation. \( x = -\sqrt{3} \) and \( x = 1 \) are extraneous.

68. \[ |x^2 + 6x| = 3x + 18 \]

First equation:
   \[ x^2 + 6x = 3x + 18 \]
   \[ x^2 + 3x - 18 = 0 \]
   \[ (x - 3)(x + 6) = 0 \]
   \[ x - 3 = 0 \Rightarrow x = 3 \]
   \[ x + 6 = 0 \Rightarrow x = -6 \]

Second equation:
   \[ -(x^2 + 6x) = 3x + 18 \]
   \[ 0 = x^2 + 9x + 18 \]
   \[ 0 = (x + 3)(x + 6) \]
   \[ 0 = x + 3 \Rightarrow x = -3 \]
   \[ 0 = x + 6 \Rightarrow x = -6 \]

The solutions to the original equation are \( x = \pm 3 \) and \( x = -6 \).

69. \[ |x + 1| = x^2 - 5 \]

First equation:
   \[ x + 1 = x^2 - 5 \]
   \[ x^2 - x - 6 = 0 \]
   \[ (x - 3)(x + 2) = 0 \]
   \[ x - 3 = 0 \Rightarrow x = 3 \]
   \[ x + 2 = 0 \Rightarrow x = -2 \]

Second equation:
   \[ -(x + 1) = x^2 - 5 \]
   \[ -x - 1 = x^2 - 5 \]
   \[ x^2 + x - 4 = 0 \]
   \[ x = \frac{-1 \pm \sqrt{17}}{2} \]

Only \( x = 3 \) and \( x = \frac{-1 - \sqrt{17}}{2} \) are solutions to the original equation. \( x = -2 \) and \( x = \frac{-1 + \sqrt{17}}{2} \) are extraneous.

70. \[ |x - 10| = x^2 - 10x \]

First equation:
   \[ x - 10 = x^2 - 10x \]
   \[ 0 = x^2 - 11x + 10 \]
   \[ 0 = (x - 10)(x - 1) \]
   \[ 0 = x - 10 \Rightarrow x = 10 \]
   \[ 0 = x - 10 \Rightarrow x = 10 \]

Second equation:
   \[ -(x - 10) = x^2 - 10x \]
   \[ 0 = x^2 - 9x - 10 \]
   \[ 0 = (x - 9)(x + 1) \]
   \[ 0 = x - 10 \Rightarrow x = 10 \]

The solutions to the original equation are \( x = 10 \) and \( x = -1 \). \( x = 1 \) is extraneous.
71. \( \frac{1}{x} - \frac{4}{x - 1} - 1 \)

(a)

(b) \( x \)-intercept: \( -1, 0 \)

(c) 
\[
0 = \frac{1}{x} - \frac{4}{x - 1} - 1 \\
0 = (x - 1) - 4x - x(x - 1) \\
0 = x - 1 - 4x - x^2 + x \\
0 = -x^2 - 2x - 1 \\
0 = x^2 + 2x + 1 \\
0 = (x + 1)^2 \\
x + 1 = 0 \implies x = -1
\]

(d) The \( x \)-intercept of the graph is the same as the solution to the equation.

72. \( \frac{9}{x + 1} - 5 \)

(a)

(b) \( x \)-intercept: \( 2, 0 \)

(c) 
\[
0 = x + \frac{9}{x + 1} - 5 \\
0 = x + \frac{9}{x + 1} - 5 \\
0 = x(x + 1) + (x + 1) \frac{9}{x + 1} - 5(x + 1) \\
0 = x^2 + x + 9 - 5x - 5 \\
0 = x^2 - 4x + 4 \\
0 = (x - 2)(x - 2) \\
0 = x - 2 \implies x = 2 \\
x = 2
\]

(d) The \( x \)-intercept and the solution are the same.

73. \( y = |x + 1| - 2 \)

(a)

(b) \( x \)-intercepts: \( 1, 0 \), \( -3, 0 \)

(c) 
\[
0 = |x + 1| - 2 \\
2 = |x + 1| \\
x + 1 = 2 \quad \text{OR} \quad -(x + 1) = 2 \\
x = 1 \quad \text{OR} \quad -x - 1 = 2 \\
-x = 3 \\
x = -3
\]

(d) The \( x \)-intercepts of the graph are the same as the solutions to the equation.

74. \( y = |x - 2| - 3 \)

(a)

(b) \( x \)-intercepts: \( 5, 0 \), \( -1, 0 \)

(c) 
\[
0 = |x - 2| - 3 \\
3 = |x - 2| \\
x - 2 = 3 \implies x = 5 \quad \text{OR} \quad -(x - 2) = 3 \\
-x + 2 = 3 \implies x = -1 \\
x = 5, -1
\]

(d) The \( x \)-intercepts and the solutions are the same.

75. \( 3.2x^4 - 1.5x^2 - 2.1 = 0 \)

\[
x^2 = \frac{1.5 \pm \sqrt{1.5^2 - 4(3.2)(-2.1)}}{2(3.2)}
\]

Using the positive value for \( x^2 \), we have \( x = \pm \sqrt{\frac{1.5 + \sqrt{29.13}}{6.4}} = \pm 1.038. \)
76. \( 7.08x^6 + 4.15x^3 - 9.6 = 0 \)

\[ a = 7.08, \quad b = 4.15, \quad c = -9.6 \]
\[ x^3 = \frac{-4.15 \pm \sqrt{(4.15)^2 - 4(-9.6)(7.08)}}{2(7.08)} \]
\[ x = \frac{-4.15 \pm \sqrt{289.0945}}{14.16} \]
\[ x = \sqrt[3]{-4.15 + \sqrt{289.0945}} = 0.968 \]
\[ x = \sqrt[3]{-4.15 - \sqrt{289.0945}} = -1.143 \]

77. \( 1.8x - 6\sqrt{x} - 5.6 = 0 \) Given equation

\[ 1.8\left(\sqrt{x}\right)^2 - 6\sqrt{x} - 5.6 = 0 \]

Use the Quadratic Formula with \( a = 1.8, b = -6, \) and \( c = -5.6. \)

\[ \sqrt{x} = \frac{6 \pm \sqrt{36 - 4(1.8)(-5.6)}}{2(1.8)} \]
\[ \sqrt{x} = \frac{6 \pm 8.7361}{3.6} \]

Considering only the positive value for \( \sqrt{x}, \) we have:

\[ \sqrt{x} = 4.0934 \]
\[ x = 16.756. \]

78. \( 4x^{2/3} + 8x^{1/3} + 3.6 = 0 \)

\[ a = 4, \quad b = 8, \quad c = 3.6 \]
\[ x^{1/3} = \frac{-8 \pm \sqrt{64 - 4(4)(3.6)}}{2(4)} \]
\[ x = \left[ \frac{-8 + \sqrt{64}}{8} \right]^3 \approx -0.320 \]
\[ x = \left[ \frac{-8 - \sqrt{64}}{8} \right]^3 \approx -2.280 \]

79. -2 and 5

One possible equation is:

\( (x - (-2))(x - 5) = 0 \)
\( (x + 2)(x - 5) = 0 \)
\( x^2 - 3x - 10 = 0 \)

Any non-zero multiple of this equation would also have these solutions.

80. 0, 3, 5

\((x - 0)(x - 3)(x - 5) = 0\)
\(x(x - 3)(x - 5) = 0\)
\(x(x^2 - 8x + 15) = 0\)
\(x^3 - 8x^2 + 15x = 0\)

Any non-zero multiple of this equation would also have these solutions.

81. \(-\frac{7}{12}\) and \(\frac{6}{15}\)

One possible equation is:

\( x = -\frac{7}{12} \Rightarrow 3x = -7 \Rightarrow 3x + 7 \) is a factor.
\( x = \frac{6}{15} \Rightarrow 7x = 6 \Rightarrow 7x - 6 \) is a factor.
\( (3x + 7)(7x - 6) = 0 \)
\( 21x^2 + 31x - 42 = 0 \)

Any non-zero multiple of this equation would also have these solutions.

82. \(-\frac{1}{4}, \quad -\frac{3}{4}\)

\((x - (-\frac{1}{4}))(x - (-\frac{3}{4})) = 0\)
\((x + \frac{1}{4})(x + \frac{3}{4}) = 0\)
\(x^2 + \frac{1}{2}x + \frac{1}{8} = 0\)
\(40x^2 + 32x + 5x + 4 = 0\)
\(40x^2 + 37x + 4 = 0\)

Any non-zero multiple of this equation would also have these solutions.

83. \(\sqrt{3}, \quad -\sqrt{3}, \quad \) and 4

One possible equation is:

\((x - \sqrt{3})(x - (-\sqrt{3}))(x - 4) = 0\)
\((x - \sqrt{3})(x + \sqrt{3})(x - 4) = 0\)
\((x^2 - 3)(x - 4) = 0\)
\(x^3 - 4x^2 - 3x + 12 = 0\)

Any non-zero multiple of this equation would also have these solutions.

84. \(2\sqrt{7}, \quad -\sqrt{7}\)

\((x - 2\sqrt{7})(x + \sqrt{7}) = 0\)
\(x^2 + x\sqrt{7} - 2x\sqrt{7} - 2(7) = 0\)
\(x^2 - x\sqrt{7} - 14 = 0\)

Any non-zero multiple of this equation would also have these solutions.

85. \(-1, \quad i, \quad \) and \(-i\)

One possible equation is:

\((x - (-1))(x - i)(x - (-i)) = 0\)
\((x + 1)(x - 1)(x - i)(x + i) = 0\)
\((x^2 - 1)(x^2 + 1) = 0\)
\(x^4 - 1 = 0\)

Any non-zero multiple of this equation would also have these solutions.
86. \((x - 4i)(x + 4i)(x - 6)(x + 6) = 0\)
\((x^2 + 16)(x^2 - 36) = 0\)
\(x^4 - 20x^2 - 576 = 0\)

87. Let \(x\) be the number of students in the original group. Then, \(\frac{1700}{x}\) is the original cost per student.

When six more students join the group, the cost per student becomes \(\frac{1700}{x} - 7.50\).

**Model:** (Cost per student) \(\cdot\) (Number of students) = Total cost
\[
\left(\frac{1700}{x} - 7.5\right)(x + 6) = 1700
\]
\((3400 - 15x)(x + 6) = 3400x\) Multiply both sides by \(2x\) to clear fractions.
\(-15x^2 - 90x + 20,400 = 0\)
\[x = \frac{90 \pm \sqrt{(-90)^2 - 4(-15)(20,400)}}{2(-15)} = \frac{90 \pm 1110}{-30}\]

Using the positive value for \(x\) we conclude that the original number was \(x = 34\) students.

88. **Model:** \(\left(\frac{\text{Cost per student}}{\text{Number of students}}\right) = \left(\frac{\text{Monthly rent}}{\text{rate}}\right)\)

**Labels:**
- Monthly rent = \(x\)
- Number of students = 4
- Original cost per student = \(\frac{x}{3}\)
- Cost per student = \(\frac{x}{3} - 75\)

**Equation:** \(\left(\frac{x}{3} - 75\right)(4) = x\)
\[\frac{4x}{3} - 300 = x\]
\[\frac{4x}{3} - x = 300\]
\[\frac{x}{3} = 300\]
\[x = 900\]

The monthly rent is $900.

89. **Model:** Time = \(\frac{\text{Distance}}{\text{Rate}}\)

**Labels:**
- Let \(x\) = average speed of the plane. Then we have a travel time of \(t = \frac{145}{x}\). If the average speed is increased by 40 mph, then

\[t - \frac{12}{60} = \frac{145}{x + 40}\]

\[t = \frac{145}{x + 40} + \frac{1}{5}\]

Now, we equate these two equations and solve for \(x\).

**Equation:** \[\frac{145}{x} = \frac{145}{x + 40} + \frac{1}{5}\]
\[145(x + 40) = 145(x) + x(x + 40)\]
\[725x + 29,000 = 725x + x^2 + 40x\]
\[0 = x^2 + 40x - 29,000\]

Using the positive value for \(x\) found by the Quadratic Formula, we have \(x = 151.5\) mph and \(x + 40 = 191.5\) mph. The airspeed required to obtain the decrease in travel time is 191.5 miles per hour.
90. **Model:**  
(Rate) \cdot (time) = (distance) 

**Labels:**  
Distance = 1080  
Original time = \( t \)  
Original rate = \( \frac{1080}{t} \)  
Return time = \( t + 2.5 \)  
Return rate = \( \frac{1080}{t} - 6 \)  

**Equation:**  
\[
\left( \frac{1080}{t} - 6 \right)(t + 2.5) = 1080
\]

91.  
\[
A = P \left( 1 + \frac{r}{n} \right)^{nt}
\]

\[
3052.49 = 2500 \left( 1 + \frac{r}{12} \right)^{12(5)}
\]

\[
1.220996 = \left( 1 + \frac{r}{12} \right)^{60}
\]

\[
(1.220996)^{1/60} = 1 + \frac{r}{12}
\]

\[
[(1.220996)^{1/60} - 1]12 = r
\]

\[
r \approx 0.04 = 4\%
\]

92. (a) Use \( A = P \left( 1 + \frac{r}{n} \right)^{nt} \).  

\[
25,000 = 10,000 \left( 1 + \frac{r}{4} \right)^{4 \cdot 20}
\]

\[
25,000 = 10,000 \left( 1 + \frac{r}{4} \right)^{80}
\]

\[
\frac{5}{2} = \left( 1 + \frac{r}{4} \right)^{80}
\]

\[
\left( \frac{5}{2} \right)^{1/80} = 1 + \frac{r}{4}
\]

\[
4 \left( \frac{5}{2} \right)^{1/80} - 1 = r \approx 0.046 \text{ or } 4.6\%
\]

(b)  
\[
32,000 = 10,000 \left( 1 + \frac{r}{4} \right)^{80}
\]

\[
\frac{16}{5} = \left( 1 + \frac{r}{4} \right)^{80}
\]

\[
\left( \frac{16}{5} \right)^{1/80} = 1 + \frac{r}{4}
\]

\[
4 \left( \frac{16}{5} \right)^{1/80} - 1 = r \approx 0.059 \text{ or } 5.9\%
\]

93.  
\[
M = 463.97 + 111.6 \sqrt{t}, \ 4 \leq t \leq 12
\]

(a)  
\[
463.97 + 111.6 \sqrt{t} = 816
\]

\[
t = \left( \frac{352.03}{111.6} \right)^2
\]

\[
t \approx 9.95
\]

The number of medical doctors reached 816,000 late during the year 1999.

(b)  
\[
463.97 + 111.6 \sqrt{t} = 900
\]

\[
t = \left( \frac{436.03}{111.6} \right)^2
\]

\[
t \approx 15.27
\]

The model predicts the number of medical doctors will reach 900,000 during the year 2005.

The actual number of medical doctors in 2005 is about 800,000.
94. (a) \( P = 200 \) million when:

\[
\begin{align*}
\frac{182.45 - 3.189t}{1.00 - 0.026t} &= 200 \\
182.45 - 3.189t &= 200(1.00 - 0.026t) \\
182.45 - 3.189t &= 200 - 5.2t \\
2.011t &= 17.55 \\
t &= 8.7
\end{align*}
\]

So the total voting-age population reached 200 million during 1998.

(b) For \( P = 230 \):

\[
\begin{align*}
\frac{182.45 - 3.189t}{1.00 - 0.026t} &= 230 \\
182.45 - 3.189t &= 230(1.00 - 0.026t) \\
182.45 - 3.189t &= 230 - 5.98t \\
2.791t &= 47.55 \\
t &= 17
\end{align*}
\]

The model predicts that the total voting-age population will reach 230 million during 2007. This value is reasonable but the model is reaching its limit since it soon begins to rise very fast due to its asymptotic behavior.

95. \( T = 75.82 - 2.11x + 43.51\sqrt{x}, 5 \leq x \leq 40 \)

(a)

<table>
<thead>
<tr>
<th>Absolute Pressure, ( x )</th>
<th>Temperature ( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>162.56</td>
</tr>
<tr>
<td>10</td>
<td>192.31</td>
</tr>
<tr>
<td>15</td>
<td>212.68</td>
</tr>
<tr>
<td>20</td>
<td>228.20</td>
</tr>
<tr>
<td>25</td>
<td>240.62</td>
</tr>
<tr>
<td>30</td>
<td>250.83</td>
</tr>
<tr>
<td>35</td>
<td>259.38</td>
</tr>
<tr>
<td>40</td>
<td>266.60</td>
</tr>
</tbody>
</table>

(b) \( T = 212^\circ \) when \( x = 15 \) pounds per square inch.

96. When \( C = 2.5 \) we have:

\[
\begin{align*}
2.5 &= \sqrt{0.2x + 1} \\
6.25 &= 0.2x + 1 \\
5.25 &= 0.2x \\
x &= 26.25 = 26,250 \text{ passengers}
\end{align*}
\]

97. \( 37.55 = 40 - \sqrt{0.01x + 1} \)

\[
\begin{align*}
\sqrt{0.01x + 1} &= 2.45 \\
0.01x + 1 &= 6.0025 \\
0.01x &= 5.0025 \\
x &= 500.25
\end{align*}
\]

Rounding \( x \) to the nearest whole unit yields \( x = 500 \) units.

98. When \( p = \$750 \), we have:

\[
\begin{align*}
750 &= 800 - \sqrt{0.01x + 1} \\
50 &= \sqrt{0.01x + 1} \\
2500 &= 0.01x + 1 \\
0.01x &= 2499 \\
x &= 249,900
\end{align*}
\]

So the demand is 249,900 units when the price is \( \$750 \).
99. **Model:** \[ \left( \text{Distance from home to 1st} \right)^2 + \left( \text{distance from 1st to 2nd} \right)^2 = \left( \text{distance from home to 2nd} \right)^2 \]

**Labels:** Distance from home to 1st = \(x\), distance from 1st to 2nd = \(x\), distance from home to 2nd = 127.5

**Equation:** \(x^2 + x^2 = (127.5)^2\)

\[2x^2 = 16,256.25\]

\[x^2 = \frac{16,256.25}{2}\]

\[x = \pm \sqrt{8,128.125} \approx \pm 90\]

The distance between bases is approximately 90 feet.

100. \(d = \sqrt{100^2 + h^2}\)

(a) ![Graph](image)

\(d = 200\) when \(h = 173\) feet.

(b) | 160 | 165 | 170 | 175 | 180 | 185 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(d)</td>
<td>188.7</td>
<td>192.9</td>
<td>197.2</td>
<td>201.6</td>
<td>205.9</td>
</tr>
</tbody>
</table>

\(d = 200\) when \(h\) is between 170 and 175 feet.

101. \(S = 8\pi\sqrt{64 + h^2}\)

(a) ![Graph](image)

When \(S = 350\), \(h = 11.4\).

(b) | 8  | 9  | 10 | 11 | 12 | 13 |
<table>
<thead>
<tr>
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<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(S)</td>
<td>284.3</td>
<td>302.6</td>
<td>321.9</td>
<td>341.8</td>
<td>362.5</td>
</tr>
</tbody>
</table>

\(S = 350\) when \(h\) is between 11 and 12 inches.

102. Let \(x\) = the number of hours required for the faster person to complete the task alone. Then \(x + 2\) = the number of hours needed for the slower person to complete the same task alone. In one hour, the faster person completes \(1/x\) of the task, the slower person \(1/(x + 2)\) of the task, and together they complete \(1/8\) of the task.

\[\frac{1}{x} + \frac{1}{x + 2} = \frac{1}{8}\]

\[8(x + 2) + 8x = x(x + 2)\]

\[8x + 16 + 8x = x^2 + 2x\]

\[0 = x^2 - 14x - 16\]

By completing the square, we have \(x = 7 \pm \sqrt{65}\). Choosing the positive value for \(x\), we have the following times for each person:

Faster person: \(7 + \sqrt{65} \approx 15\) hours; slower person: \(9 + \sqrt{65} \approx 17\) hours.
103. Model: 

\[
\left( \frac{\text{Portion done}}{\text{by first person}} \right) + \left( \frac{\text{portion done}}{\text{by second person}} \right) = \left( \frac{\text{work done}}{\text{done}} \right)
\]

Labels: Work done = 1, rate of first person = \( \frac{1}{r} \), time worked by first person = 12,

rate of second person = \( \frac{1}{r + 3} \), time worked by second person = 12

Equation: 

\[
\frac{12}{r} + \frac{12}{r + 3} = 1
\]

\[
r(r + 3)\frac{12}{r} + r(r + 3)\frac{12}{r + 3} = r(r + 3)
\]

\[
12r + 36 + 12r = r^2 + 3r
\]

\[
0 = r^2 - 21r - 36
\]

\[
r = \frac{-(-21) \pm \sqrt{(-21)^2 - 4(1)(-36)}}{2(1)} = \frac{21 \pm \sqrt{385}}{2}
\]

\[
r = 23 \quad (\text{Choose the positive value for } r.)
\]

It would take approximately 23 hours and 26 hours individually.

104. (a) Verbal Model: Total cost = Cost underwater \cdot Distance underwater + Cost overland \cdot Distance overland

Labels: Total cost: \( C \)

Cost overland: $24 per foot
Distance overland in feet: 5280(8 - x)
Cost underwater: $30 per foot
Distance underwater in feet: 5280 \( \sqrt{x^2 + (3/4)^2} = 1320 \sqrt{16x^2 + 9} \)

Equation: 

\[
C = 24[5280(8 - x)] + 30(1320 \sqrt{16x^2 + 9})
\]

\[
C = 126,720(8 - x) + 39,600 \sqrt{16x^2 + 9}
\]

(b) \( C = 126,720(8 - 3) + 39,600 \sqrt{16(3)^2 + 9} \)

\[
= 126,720(5) + 39,600 \sqrt{153}
\]

\[
= 633,600 + 489,824.95
\]

\[
= 1,123,424.95
\]

So, the total cost when \( x = 3 \) is $1,123,424.95.

(c) \[
1,098,662.40 = 126,720(8 - x) + 39,600 \sqrt{16x^2 + 9}
\]

\[
1,098,662.40 = 7920[16(8 - x) + 5 \sqrt{16x^2 + 9}]
\]

\[
138.72 = 16(8 - x) + 5 \sqrt{16x^2 + 9}
\]

\[
138.72 = 128 - 16x + 5 \sqrt{16x^2 + 9}
\]

\[
16x + 10.72 = 5 \sqrt{16x^2 + 9}
\]

\[
(16x + 10.72)^2 = (5 \sqrt{16x^2 + 9})^2
\]

\[
256x^2 + 343.04x + 114.9184 = 25(16x^2 + 9)
\]

\[
256x^2 + 343.04x + 114.9184 = 400x^2 + 225
\]

\[
0 = 144x^2 - 343.04x + 110.0816
\]

By the Quadratic Formula, we have \( x = 2 \) miles or \( x = 0.382 \) mile.

—CONTINUED—
104. —CONTINUED—

(e) The cost is a minimum when $x = 1$. (The minimum cost is $1,085,040.00$.)

105. 

\[ v = \sqrt{\frac{gR}{\mu s}} \]

\[ v^2 = \frac{gR}{\mu s} \]

\[ v^2 \mu s = gR \]

\[ \frac{v^2 \mu s}{R} = g \]

106. 

\[ i = \pm \sqrt{\frac{1}{LC} \sqrt{Q^2 - q}} \]

\[ i^2 = \left( \pm \sqrt{\frac{1}{LC} \sqrt{Q^2 - q}} \right)^2 \]

\[ i^2 = \frac{1}{LC}(Q^2 - q) \]

\[ LCI^2 + q = Q^2 \]

\[ \pm \sqrt{LCI^2 + q} = Q \]

107. False—See Example 7 on page 137.

108. False. $|x| = 0$ has only one solution to check, 0.

109. The distance between $(1, 2)$ and $(x, -10)$ is 13.

\[ \sqrt{(x - 1)^2 + (-10 - 2)^2} = 13 \]

\[ (x - 1)^2 + (-12)^2 = 13^2 \]

\[ x^2 - 2x + 1 + 144 = 169 \]

\[ x^2 - 2x - 24 = 0 \]

\[ (x + 4)(x - 6) = 0 \]

\[ x + 4 = 0 \Rightarrow x = -4 \]

\[ x - 6 = 0 \Rightarrow x = 6 \]

Both $(-4, -10)$ and $(6, -10)$ are a distance of 13 from $(1, 2)$.

110. The distance between $(-8, 0)$ and $(x, 5)$ is 13.

\[ \sqrt{(x + 8)^2 + (5 - 0)^2} = 13 \]

\[ (x + 8)^2 + 5^2 = 13^2 \]

\[ x^2 + 16x + 64 + 25 = 169 \]

\[ x^2 + 16x - 80 = 0 \]

\[ (x + 20)(x - 4) = 0 \]

\[ x + 20 = 0 \Rightarrow x = -20 \]

\[ x - 4 = 0 \Rightarrow x = 4 \]

Both $(-20, 5)$ and $(4, 5)$ are a distance of 13 from $(-8, 0)$.

111. The distance between $(0, 0)$ and $(8, y)$ is 17.

\[ \sqrt{(8 - 0)^2 + (y - 0)^2} = 17 \]

\[ (8)^2 + (y)^2 = 17^2 \]

\[ 64 + y^2 = 289 \]

\[ y^2 = 225 \]

\[ y = \pm \sqrt{225} \]

\[ y = \pm 15 \]

Both $(8, 15)$ and $(8, -15)$ are a distance of 17 from $(0, 0)$.

112. The distance between $(-8, 4)$ and $(7, y)$ is 17.

\[ \sqrt{(7 + 8)^2 + (y - 4)^2} = 17 \]

\[ (15)^2 + (y - 4)^2 = 17^2 \]

\[ 225 + (y - 4)^2 = 289 \]

\[ (y - 4)^2 = 64 \]

\[ y - 4 = \pm 8 \]

\[ y = -4 \pm 8 = -4, 12 \]

Both $(7, -4)$ and $(7, 12)$ are a distance of 17 from $(-8, 4)$. 
113. \(9 + |9 - a| = b\)
\[|9 - a| = b - 9\]
\[9 - a = b - 9 \quad \text{OR} \quad 9 - a = -(b - 9)\]
\[-a = b - 18 \quad 9 - a = -b + 9\]
\[a = 18 - b \quad -a = -b\]
\[a = b\]

Thus, \(a = 18 - b\) or \(a = b\). From the original equation we know that \(b \geq 9\).

Some possibilities are:
- \(b = 9, a = 9\)
- \(b = 10, a = 8\) or \(a = 10\)
- \(b = 11, a = 7\) or \(a = 11\)
- \(b = 12, a = 6\) or \(a = 12\)
- \(b = 13, a = 5\) or \(a = 13\)
- \(b = 14, a = 4\) or \(a = 14\)

115. \(20 + \sqrt{20 - a} = b\)
\[\sqrt{20 - a} = b - 20\]
\[20 - a = b^2 - 40b + 400\]
\[-a = b^2 - 40b + 380\]
\[a = -b^2 + 40b - 380\]

This formula gives the relationship between \(a\) and \(b\). From the original equation we know that \(a \leq 20\) and \(b \geq 20\). Choose a \(b\) value, where \(b \geq 20\) and then solve for \(a\), keeping in mind that \(a \leq 20\).

Some possibilities are:
- \(b = 20, a = 20\)
- \(b = 21, a = 19\)
- \(b = 22, a = 16\)
- \(b = 23, a = 11\)
- \(b = 24, a = 4\)
- \(b = 25, a = -5\)

117. \(\frac{8}{3x} + \frac{3}{2x} = \frac{16}{6x} + \frac{9}{6x} = \frac{25}{6x}\)

118. \(\frac{2}{x^2 - 4} - \frac{1}{x^2 - 3x + 2} = \frac{2}{(x + 2)(x - 2)} - \frac{1}{(x - 1)(x - 2)}\)
\[= \frac{2(x - 1) - (x + 2)}{(x - 1)(x + 2)(x - 2)}\]
\[= \frac{2x - 2 - x - 2}{(x - 1)(x^2 - 4)}\]
\[= \frac{x - 4}{(x - 1)(x^2 - 4)}\]

119. \(\frac{2}{z + 2} - \left(\frac{3 - \frac{2}{z}}{z}\right) = \frac{2}{z + 2} - 3 + \frac{2}{z}\)
\[= \frac{2z - 3(z + 2) + 2(z + 2)}{z(z + 2)}\]
\[= \frac{2z - 3z^2 - 6z + 2z + 4}{z(z + 2)}\]
\[= \frac{-3z^2 - 2z + 4}{z(z + 2)}\]
120. \[ 25y^2 + \frac{xy}{5} = \frac{25y^2}{1} \cdot \frac{5}{xy} \]
\[ = \frac{125y}{x}, \ y \neq 0 \]

121. \[ x^2 - 22x + 121 = 0 \]
\[ (x - 11)^2 = 0 \]
\[ x - 11 = 0 \]
\[ x = 11 \]

122. \[ x(x - 20) + 3(x - 20) = 0 \]
\[ (x + 3)(x - 20) = 0 \]
\[ x + 3 = 0 \implies x = -3 \]
\[ x - 20 = 0 \implies x = 20 \]

Section 1.7  Linear Inequalities in One Variable

■ You should know the properties of inequalities.
(a) Transitive: \(a < b\) and \(b < c\) implies \(a < c\).
(b) Addition: \(a < b\) and \(c < d\) implies \(a + c < b + d\).
(c) Adding or Subtracting a Constant: \(a < b\) if \(a + c < b + c\) if \(a < b\).
(d) Multiplying or Dividing a Constant: For \(a < b,\)
\[ 1. \text{ If } c > 0, \text{ then } ac < bc \text{ and } \frac{a}{c} < \frac{b}{c}. \]
\[ 2. \text{ If } c < 0, \text{ then } ac > bc \text{ and } \frac{a}{c} > \frac{b}{c}. \]

■ You should be able to solve absolute value inequalities.
(a) \(|x| < a\) if and only if \(-a < x < a\).
(b) \(|x| > a\) if and only if \(x < -a\) or \(x > a\).

Vocabulary Check

1. solution set  2. graph  3. negative
4. equivalent  5. double  6. union

1. Interval: \([-1, 5]\)
   (a) Inequality: \(-1 \leq x \leq 5\)
   (b) The interval is bounded.

2. Interval: \((2, 10]\)
   (a) Inequality: \(2 < x \leq 10\)
   (b) The interval is bounded.

3. Interval: \((11, \infty)\)
   (a) Inequality: \(x > 11\)
   (b) The interval is unbounded.

4. Interval: \([-5, \infty)\)
   (a) Inequality: \(-5 \leq x < \infty\) or \(x \geq -5\)
   (b) The interval is unbounded.

5. Interval: \((-\infty, -2)\)
   (a) Inequality: \(x < -2\)
   (b) The interval is unbounded.

6. Interval: \((-\infty, 7]\)
   (a) Inequality: \(-\infty < x \leq 7\) or \(x < -7\)
   (b) The interval is unbounded.

7. \(x < 3\)  Matches (b).

8. \(x \geq 5\)  Matches (f).

9. \(-3 < x \leq 4\)  Matches (d).

10. \(0 \leq x \leq \frac{5}{2}\)  Matches (c).
11. \(|x| < 3 \Rightarrow -3 < x < 3\) 
   Matches (e).

12. \(|x| > 4 \Rightarrow x > 4\) or \(x < -4\) 
   Matches (a).

13. \(5x - 12 > 0\)
   \[(a) \ x = 3\]
   \[5(3) - 12 > 0\]
   \[15 > 0\]
   Yes, \(x = 3\) is a solution.

   \[(b) \ x = -3\]
   \[5(-3) - 12 > 0\]
   \[-27 > 0\]
   No, \(x = -3\) is not a solution.

   \[(c) \ x = \frac{5}{2}\]
   \[5\left(\frac{5}{2}\right) - 12 > 0\]
   \[\frac{5}{2} > 0\]
   Yes, \(x = \frac{5}{2}\) is a solution.

   \[(d) \ x = \frac{3}{2}\]
   \[5\left(\frac{3}{2}\right) - 12 > 0\]
   \[-\frac{9}{2} > 0\]
   No, \(x = \frac{3}{2}\) is not a solution.

14. \(2x + 1 < -3\)
   \[(a) \ x = 0\]
   \[2(0) + 1 < -3\]
   \[1 \neq -3\]
   No, \(x = 0\) is not a solution.

   \[(b) \ x = -\frac{1}{2}\]
   \[2\left(-\frac{1}{2}\right) + 1 < -3\]
   \[0 < -3\]
   No, \(x = -\frac{1}{2}\) is not a solution.

   \[(c) \ x = -4\]
   \[2(-4) + 1 < -3\]
   \[-7 < -3\]
   Yes, \(x = -4\) is a solution.

   \[(d) \ x = -\frac{5}{2}\]
   \[2\left(-\frac{5}{2}\right) + 1 < -3\]
   \[-4 < -3\]
   No, \(x = -\frac{5}{2}\) is not a solution.

15. \(0 < \frac{x - 2}{4} < 2\)
   \[(a) \ x = 4\]
   \[0 < 4 - 2 \leq 2\]
   \[0 < 2 \leq 2\]
   Yes, \(x = 4\) is a solution.

   \[(b) \ x = 10\]
   \[0 < 10 - 2 \leq 2\]
   \[0 < 8 \leq 2\]
   No, \(x = 10\) is not a solution.

   \[(c) \ x = 0\]
   \[0 < 0 - 2 \leq 2\]
   \[0 < -2 \leq 2\]
   No, \(x = 0\) is not a solution.

   \[(d) \ x = \frac{7}{2}\]
   \[0 < \frac{(7/2) - 2}{4} \leq 2\]
   \[0 < \frac{3}{8} \leq 2\]
   Yes, \(x = \frac{7}{2}\) is a solution.

16. \(-1 < \frac{3 - x}{2} \leq 1\)
   \[(a) \ x = 0\]
   \[-1 < \frac{3 - 0}{2} \leq 1\]
   \[-1 < \frac{3}{2} \leq 1\]
   No, \(x = 0\) is not a solution.

   \[(b) \ x = -5\]
   \[-1 < \frac{3 - (-5)}{2} \leq 1\]
   \[-1 < \frac{8}{2} \leq 1\]
   No, \(x = -5\) is not a solution.

   \[(c) \ x = 1\]
   \[-1 < \frac{3 - 1}{2} \leq 1\]
   \[-1 < \frac{1}{2} \leq 1\]
   Yes, \(x = 1\) is a solution.

   \[(d) \ x = 5\]
   \[-1 < \frac{3 - 5}{2} \leq 1\]
   \[-1 < -1 \leq 1\]
   No, \(x = 5\) is not a solution.

17. \(|x - 10| \geq 3\)
   \[(a) \ x = 13\]
   \[|13 - 10| \geq 3\]
   \[3 \geq 3\]
   Yes, \(x = 13\) is a solution.

   \[(b) \ x = -1\]
   \[|-1 - 10| \geq 3\]
   \[11 \geq 3\]
   Yes, \(x = -1\) is a solution.

   \[(c) \ x = 14\]
   \[|14 - 10| \geq 3\]
   \[4 \geq 3\]
   Yes, \(x = 14\) is a solution.

   \[(d) \ x = 9\]
   \[|9 - 10| \geq 3\]
   \[1 \geq 3\]
   No, \(x = 9\) is not a solution.
18. \( |2x - 3| < 15 \)
   \( (a) \quad x = -6 \)
   \[ |2(-6) - 3| < 15 \]
   \[ 15 < 15 \]
   No, \( x = -6 \) is not a solution.
   \( (b) \quad x = 0 \)
   \[ |2(0) - 3| < 15 \]
   \[ 3 < 15 \]
   Yes, \( x = 0 \) is a solution.
   \( (c) \quad x = 12 \)
   \[ |2(12) - 3| < 15 \]
   \[ 21 < 15 \]
   No, \( x = 12 \) is not a solution.
   \( (d) \quad x = 7 \)
   \[ |2(7) - 3| < 15 \]
   \[ 11 < 15 \]
   Yes, \( x = 7 \) is a solution.

19. \( 4x < 12 \)
   \[ \frac{1}{2}(4x) < \frac{1}{2}(12) \]
   \[ x < 3 \]

20. \( 10x < -40 \)
   \[ x < -4 \]

21. \( -2x > -3 \)
   \[ -\frac{2}{3}(-2x) < (-\frac{2}{3})(-3) \]
   \[ x < \frac{3}{2} \]

22. \( -6x > 15 \)
   \[ x < -\frac{15}{6} \text{ or } x < -\frac{5}{2} \]

23. \( x - 5 \geq 7 \)
   \[ x \geq 12 \]

24. \( x + 7 \leq 12 \)
   \[ x \leq 5 \]

25. \( 2x + 7 < 3 + 4x \)
   \[ -2x < -4 \]
   \[ x > 2 \]

26. \( 3x + 1 \geq 2 + x \)
   \[ 2x \geq 1 \]
   \[ x \geq \frac{1}{2} \]

27. \( 2x - 1 \geq 1 - 5x \)
   \[ 7x \geq 2 \]
   \[ x \geq \frac{2}{7} \]

28. \( 6x - 4 \leq 2 + 8x \)
   \[ -2x \leq 6 \]
   \[ x \geq -3 \]

29. \( 4 - 2x < 3(3 - x) \)
   \[ 4 - 2x < 9 - 3x \]
   \[ x < 5 \]

30. \( 4(x + 1) < 2x + 3 \)
   \[ 4x + 4 < 2x + 3 \]
   \[ 2x < -1 \]
   \[ x < -\frac{1}{2} \]

31. \( \frac{1}{2}x - 6 \leq x - 7 \)
   \[ -\frac{1}{2}x \leq -1 \]
   \[ x \geq 4 \]

32. \( 3 + \frac{7}{2}x > x - 2 \)
   \[ 21 + 2x > 7x - 14 \]
   \[ -5x > -35 \]
   \[ x < 7 \]

33. \( \frac{1}{2}(8x + 1) \geq 3x + \frac{5}{2} \)
   \[ 4x + \frac{1}{2} \geq 3x + \frac{5}{2} \]
   \[ x \geq 2 \]
34. \[9x - 1 < \frac{3}{2}(16x - 2)\]
\[36x - 4 < 48x - 6\]
\[-12x < -2\]
\[x > \frac{1}{6}\]

35. \[3.6x + 11 \geq -3.4\]
\[3.6x \geq -14.4\]
\[x \geq -4\]

36. \[15.6 - 1.3x < -5.2\]
\[-1.3x < -20.8\]
\[x > 16\]

37. \[1 < 2x + 3 < 9\]
\[-2 < 2x < 6\]
\[-1 < x < 3\]

38. \[-8 \leq -(3x + 5) < 13\]
\[-8 \leq -3x - 5 < 13\]
\[-3 \leq -3x < 18\]
\[-6 < x \leq 6\]

39. \[-4 < \frac{2x - 3}{3} < 4\]
\[-12 < 2x - 3 < 12\]
\[-9 < 2x < 15\]
\[-\frac{9}{2} < x < \frac{15}{2}\]

40. \[0 \leq \frac{x + 3}{2} < 5\]
\[0 \leq x + 3 < 10\]
\[-3 \leq x < 7\]

41. \[\frac{3}{4} > x + 1 > \frac{1}{4}\]
\[\frac{1}{4} > x > -\frac{3}{4}\]
\[\frac{3}{4} < x < -\frac{1}{4}\]

42. \[-1 < 2 - \frac{x}{3} < 1\]
\[-3 < 6 - x < 3\]
\[-9 < -x < -3\]
\[3 < x < 9\]

43. \[3.2 \leq 0.4x - 1 \leq 4.4\]
\[4.2 \leq 0.4x \leq 5.4\]
\[10.5 \leq x \leq 13.5\]

44. \[4.5 > \frac{1.5x + 6}{2} > 10.5\]
\[9 > 1.5x + 6 > 21\]
\[3 > 1.5x > 15\]
\[2 > x > 10\]

There is no solution.

45. \[|x| < 6\]
\[-6 < x < 6\]

46. \[|x| > 4\]
\[x < -4 \text{ or } x > 4\]

47. \[\frac{|x|}{2} > 1\]
\[\frac{x}{2} < -1 \text{ or } \frac{x}{2} > 1\]
\[x < -2 \text{ or } x > 2\]

48. \[\left|\frac{x}{5}\right| > 3\]
\[\frac{x}{5} < -3 \text{ or } \frac{x}{5} > 3\]
\[x < -15 \text{ or } x > 15\]

49. \[|x - 5| < -1\]

No solution. The absolute value of a number cannot be less than a negative number.

50. There is no solution because the absolute value of a number cannot be less than a negative number.
51. \(|x - 20| \leq 6\)
-6 \leq x - 20 \leq 6
14 \leq x \leq 26

52. \(|x - 8| \geq 0\)
-8 \geq x - 8 \geq 0
x \geq 8
-x \geq -8
x \leq 8

All real numbers x

53. \(|3 - 4x| \geq 9\)
3 - 4x \leq -9 \quad \text{or} \quad 3 - 4x \geq 9
-4x \leq -12 \quad -4x \geq 6
x \geq 3 \quad x \leq -\frac{3}{2}

54. \(|1 - 2x| < 5\)
-5 < 1 - 2x < 5
-6 < -2x < 4
3 > x > -2
-2 < x < 3

55. \(|\frac{x - 3}{2}| \geq 4\)
\frac{x - 3}{2} \leq -4 \quad \text{or} \quad \frac{x - 3}{2} \geq 4
x - 3 \leq -8 \quad x - 3 \geq 8
x \leq -5 \quad x \geq 11

56. \(|1 - \frac{2x}{3}| < 1\)
-1 < 1 - \frac{2x}{3} < 1
-2 < -\frac{2x}{3} < 0
3 > x > 0
0 < x < 3

57. \(|9 - 2x| - 2 < -1\)
|9 - 2x| < 1
-1 < 9 - 2x < 1
-10 < -2x < -8
5 > x > 4
4 < x < 5

58. \(|x + 14| + 3 > 17\)
|x + 14| > 14
x + 14 < -14 \quad \text{or} \quad x + 14 > 14
x < -28
x > 0

59. \(2|x + 10| \geq 9\)
|x + 10| \geq \frac{9}{2}
-\frac{9}{2} \leq x + 10 \leq \frac{9}{2}
x \leq -\frac{29}{2} \quad x \geq -\frac{11}{2}

60. \(3|4 - 5x| \leq 9\)
|4 - 5x| \leq \frac{3}{2}
-\frac{3}{2} \leq 4 - 5x \leq \frac{3}{2}
-\frac{7}{5} \leq x \leq \frac{1}{5}
\frac{7}{5} \leq x \leq \frac{3}{5}
61. $6x > 12$
   
   $x > 2$

62. $3x - 1 \leq 5$
   
   $3x \leq 6$
   
   $x \leq 2$

63. $5 - 2x \geq 1$
   
   $-2x \geq -4$
   
   $x \leq 2$

64. $3(x + 1) < x + 7$
   
   $3x + 3 < x + 7$
   
   $2x < 4$
   
   $x < 2$

65. $|x - 8| \leq 14$
   
   $-14 \leq x - 8 \leq 14$
   
   $-6 \leq x \leq 22$

66. $|2x + 9| > 13$
   
   $2x + 9 < -13$ or $2x + 9 > 13$
   
   $2x < -22$  $2x > 4$
   
   $x < -11$  $x > 2$

67. $2|x + 7| \geq 13$
   
   $|x + 7| \geq \frac{13}{2}$
   
   $x + 7 \leq -\frac{13}{2}$ or $x + 7 \geq \frac{13}{2}$
   
   $x \leq -\frac{27}{2}$  $x \geq -\frac{1}{2}$

68. $\frac{1}{2}|x + 1| \leq 3$
   
   $|x + 1| \leq 6$
   
   $-6 \leq x + 1 \leq 6$
   
   $-7 \leq x \leq 5$

69. $y = 2x - 3$
   
   (a) $y \geq 1$
   
   $2x - 3 \geq 1$
   
   $2x \geq 4$
   
   $x \geq 2$
   
   (b) $y \leq 0$
   
   $2x - 3 \leq 0$
   
   $2x \leq 3$
   
   $x \leq \frac{3}{2}$

70. $y = \frac{3}{2}x + 1$
   
   (a) $y \leq 5$
   
   $\frac{3}{2}x + 1 \leq 5$
   
   $\frac{3}{2}x \leq 4$
   
   $x \leq 6$
   
   (b) $y \geq 0$
   
   $\frac{3}{2}x + 1 \geq 0$
   
   $\frac{3}{2}x \geq -1$
   
   $x \geq -\frac{3}{2}$
71. \( y = -\frac{1}{2}x + 2 \)
   (a) \( 0 \leq y \leq 3 \)
       \[ 0 \leq -\frac{1}{2}x + 2 \leq 3 \]
       \[-2 \leq -\frac{1}{2}x \leq 1 \]
       \[4 \geq x \geq -2 \]
   (b) \( y \geq 0 \)
       \[-\frac{1}{2}x + 2 \geq 0 \]
       \[-\frac{1}{2}x \geq -2 \]
       \[x \leq 4 \]

72. \( y = -3x + 8 \)
   (a) \(-1 \leq y \leq 3 \)
       \[-1 \leq -3x + 8 \leq 3 \]
       \[-9 \leq -3x \leq -5 \]
       \[3 \geq x \geq \frac{5}{3} \]
   (b) \( y \leq 0 \)
       \[-3x + 8 \leq 0 \]
       \[-3x \leq -8 \]
       \[x \geq \frac{8}{3} \]

73. \( y = |x - 3| \)
   (a) \( y \leq 2 \)
       \[|x - 3| \leq 2 \]
       \[-2 \leq x - 3 \leq 2 \]
       \[1 \leq x \leq 5 \]
   (b) \( y \geq 4 \)
       \[|x - 3| \geq 4 \]
       \[x - 3 \leq -4 \text{ or } x - 3 \geq 4 \]
       \[x \leq -1 \text{ or } x \geq 7 \]

74. \( y = \left| \frac{1}{2}x + 1 \right| \)
   (a) \( y \leq 4 \)
       \[\left| \frac{1}{2}x + 1 \right| \leq 4 \]
       \[-4 \leq \frac{1}{2}x + 1 \leq 4 \]
       \[-5 \leq \frac{1}{2}x \leq 3 \]
       \[-10 \leq x \leq 6 \]
   (b) \( y \geq 1 \)
       \[\left| \frac{1}{2}x + 1 \right| \geq 1 \]
       \[\frac{1}{2}x + 1 \leq -1 \text{ or } \frac{1}{2}x + 1 \geq 1 \]
       \[\frac{1}{2}x \leq -2 \text{ or } \frac{1}{2}x \geq 0 \]
       \[x \leq -4 \text{ or } x \geq 0 \]

75. \( x - 5 \geq 0 \)
   \[x \geq 5 \]
   \[\left[5, \infty\right) \]

76. \( \sqrt{x - 10} \)
   \[x - 10 \geq 0 \]
   \[x \geq 10 \]
   \[\left[10, \infty\right) \]

77. \( x + 3 \geq 0 \)
   \[x \geq -3 \]
   \[\left[-3, \infty\right) \]

78. \( \sqrt[3]{-x} \)
   \[3 - x \geq 0 \]
   \[3 \geq x \]
   \[\left(-\infty, 3\right] \]

79. \( 7 - 2x \geq 0 \)
   \[-2x \geq -7 \]
   \[x \leq \frac{7}{2} \]
   \[\left(-\infty, \frac{7}{2}\right] \]

80. \( \sqrt[3]{6x + 15} \)
   \[6x + 15 \geq 0 \]
   \[6x \geq -15 \]
   \[x \geq -\frac{5}{2} \]
   \[\left[-\frac{5}{2}, \infty\right) \]

81. \( |x - 10| < 8 \)
   All real numbers within 8 units of 10.

82. \( |x - 8| > 4 \)
   All real numbers more than 4 units from 8

83. The midpoint of the interval \([-3, 3]\) is 0. The interval represents all real numbers \(x\) no more than 3 units from 0.
   \[|x - 0| \leq 3 \]
   \[|x| \leq 3 \]

84. The graph shows all real numbers more than 3 units from 0.
   \[|x - 0| > 3 \]
   \[|x| > 3 \]

85. The graph shows all real numbers at least 3 units from 7.
   \[|x - 7| \geq 3 \]
86. The graph shows all real numbers no more than 4 units from \(-1\).
\[ |x + 1| \leq 4 \]

87. All real numbers within 10 units of 12
\[ |x - 12| < 10 \]

88. All real numbers at least 5 units from 8
\[ |x - 8| \geq 5 \]

89. All real numbers more than 4 units from \(-3\)
\[ |x - (-3)| > 4 \]
\[ |x + 3| > 4 \]

90. All real numbers no more than 7 units from \(-6\)
\[ |x + 6| \leq 7 \]

91. Let \(x\) = the number of checks written in a month.

Type A account charges: \(6.00 + 0.25x\)
Type B account charges: \(4.50 + 0.50x\)
\[ 6.00 + 0.25x < 4.50 + 0.50x \]
\[ 1.50 < 0.25x \]
\[ 6 < x \]

If you write more than six checks a month, then the charges for the type A account are less than the charges for the type B account.

92. \(3000 + 0.03x \leq 0.1x\)
\[ 3000 < 0.07x \]
\[ 42,857 < x \]
You must make more than 42,857 copies to justify buying the copier.

93. \(1000(1 + r(2)) > 1062.50\)
\[ 1 + 2r > 1.0625 \]
\[ 2r > 0.0625 \]
\[ r > 0.03125 \]
\[ r > 3.125\% \]

94. \(825 < 750(1 + r(2))\)
\[ 825 < 750(1 + 2r) \]
\[ 825 < 750 + 1500r \]
\[ 75 < 1500r \]
\[ 0.05 < r \]
The rate must be more than 5%.

95. \(R > C\)
\[ 115.95x > 95x + 750 \]
\[ 20.95x > 750 \]
\[ x > 35.7995 \]
\[ x \geq 36 \text{ units} \]

96. \(24.55x > 15.4x + 150,000\)
\[ 9.15 > 150,000 \]
\[ x > 16,393.44262 \]
Because the number of units \(x\) must be an integer, the product will return a profit when at least 16,394 units are sold.

97. Let \(x\) = daily sales level (in dozens) of doughnuts.
\[ \text{Revenue: } R = 2.95x \]
\[ \text{Cost: } C = 150 + 1.45x \]
\[ \text{Profit: } P = R - C \]
\[ = 2.95x - (150 + 1.45x) \]
\[ = 1.50x - 150 \]
\[ 50 \leq P \leq 200 \]
\[ 50 \leq 1.50x - 150 \leq 200 \]
\[ 200 \leq 1.50x \leq 350 \]
\[ 133\frac{1}{3} \leq x \leq 233\frac{1}{3} \]
In whole dozens, \(134 \leq x \leq 234\).

98. The goal is to lose \(164 - 128 = 36\) pounds. At \(1\frac{1}{2}\) pounds per week, it will take 24 weeks.
\[ 36 \div 1\frac{1}{2} = 36 \times 2 \]
\[ = 12 \times 2 \]
\[ = 24 \]
99. (a) \( y = 0.067x - 5.638 \)

(b) From the graph we see that \( y \geq 3 \) when \( x \geq 129 \).
Algebraically we have:

\[
3 \leq 0.067x - 5.638 \\
8.638 \leq 0.067x \\
x \geq 129
\]

IQ scores are not a good predictor of GPAs. Other factors include study habits, class attendance, and attitude.

100. (a) and (b)

\[
\begin{array}{cccccc}
 x & 165 & 180 & 185 & 200 & 210 \\
y & 170 & 185 & 200 & 255 & 205 \\
1.3x - 36 & 179 & 203 & 159 & 237 & 219 \\
\end{array}
\]

(c) One estimate is \( x \geq 181 \) pounds.

(d) \( 1.3x - 36 \geq 200 \)

\[
1.3x \geq 236 \\
x \geq 181.5385 \approx 181.54 \text{ pounds}
\]

101. \( S = 1.05t + 31.0 \), \( 0 \leq t \leq 12 \)

(a) \( 32 \leq 1.05t + 31 \leq 42 \)

\[
1 \leq 1.05t \leq 11 \\
0.95 \leq t \leq 10.48
\]

Rounding to the nearest year, \( 1 \leq t \leq 10 \). The average salary was at least $32,000 but not more than $42,000 between 1991 and 2000.

(b) \( 1.05t + 31 > 48 \)

\[
1.05t > 17 \\
t > 16
\]

According to the model, the average salary will exceed $48,000 in 2006.

102. (a) \( 70 \leq 1.64t + 67.2 \leq 80 \)

\[
2.8 \leq 1.64t \leq 12.8 \\
1.7 \leq t \leq 7.8
\]

The number of eggs produced was between 70 and 80 billion from late 1991 until late 1997.

(b) \( E = 1.64t + 67.2 \geq 95 \) when \( t \geq 16.95 \).

So the annual egg production will exceed 95 billion in 2007.

103. \( |s - 10.4| \leq 0.16 \)

\[
-0.16 \leq s - 10.4 \leq 0.16 \\
-0.0625 \leq s - 10.4 \leq 0.0625 \\
10.3375 \leq s \leq 10.4625
\]

Since \( A = s^2 \), we have

\[
(10.3375)^2 \leq \text{area} \leq (10.4625)^2 \\
106.864 \leq \text{area} \leq 109.464
\]

104. \( 24.2 - 0.25 \leq s \leq 24.2 + 0.25 \)

\[
23.95 \leq s \leq 24.45
\]

The interval containing the possible side lengths \( s \) in centimeters of the square is \([23.95, 24.45]\), so the interval containing the possible areas in square centimeters is \([23.95^2, 24.45^2]\), or \([573.6025, 597.8025]\).
105. \[ |x - 15| \leq \frac{1}{10} \]
   \[-\frac{1}{10} \leq x - 15 \leq \frac{1}{10} \]
   \[14.9 \leq x \leq 15.1 \text{ gallons} \]
   
   \[\frac{1}{10} (\$1.89) = \$0.19 \]
   
   You might have been undercharged or overcharged by $0.19.

107. \[ \left| \frac{t - 15.6}{1.9} \right| < 1 \]
   
   \[-1 < \frac{t - 15.6}{1.9} < 1 \]
   
   \[-1.9 < t - 15.6 < 1.9 \]
   
   \[13.7 < t < 17.5 \]
   
   Two-thirds of the workers could perform the task in the time interval between 13.7 minutes and 17.5 minutes.

109. \[ |h - 50| \leq 30 \]
   
   \[-30 \leq h - 50 \leq 30 \]
   
   \[20 \leq h \leq 80 \]
   
   The minimum relative humidity is 20 and the maximum is 80.

111. False. If \( c \) is negative, then \( ac \geq bc \).

112. False. If \(-10 \leq x \leq 8\), then \(-10 \geq -x \) and \(-x \geq -8\).

113. \[ |x - a| \geq 2 \]

   Matches (b).
   
   \[ x - a \leq -2 \]
   
   \[ x \leq a - 2 \] or \[ x - a \geq 2 \]
   
   \[ x \geq a + 2 \]

114. \[ |ax - b| \leq c \Rightarrow c \text{ must be greater than or equal to zero.} \]

   \[-c \leq ax - b \leq c \]

   \[b - c \leq ax \leq b + c \]

   Let \( a = 1 \), then \( b - c = 0 \) and \( b + c = 10 \). This is true when \( b = c = 5 \).

   One set of values is \( a = 1, b = 5, c = 5 \).

   (Note: This solution is not unique. Any positive multiple of these values will also work, such as \( a = 2, b = c = 10 \) or \( a = 3, b = c = 15 \).)

115. \((-4, 2)\) and \((1, 12)\)

   \[ d = \sqrt{(1 - (-4))^2 + (12 - 2)^2} = \sqrt{3^2 + 10^2} = \sqrt{125} = 5 \sqrt{5} \]

   Midpoint: \( \left( \frac{-4 + 1}{2}, \frac{2 + 12}{2} \right) = \left( \frac{-3}{2}, 7 \right) \)
116. \( d = \sqrt{(1-10)^2 + (-2 -3)^2} = \sqrt{(-9)^2 + (-5)^2} = \sqrt{81 + 25} = \sqrt{106} \)

Midpoint: \( \left( \frac{1 + 10}{2}, \frac{-2 + 3}{2} \right) = \left( \frac{11}{2}, \frac{1}{2} \right) \)

117. (3, 6) and (-5, -8)

\( d = \sqrt{(-5 - 3)^2 + (-8 - 6)^2} = \sqrt{(-8)^2 + (-14)^2} = \sqrt{260} = 2\sqrt{65} \)

Midpoint: \( \left( \frac{3 + (-5)}{2}, \frac{6 + (-8)}{2} \right) = (-1, -1) \)

118. \( d = \sqrt{[0 - (-6)]^2 + (-3 - 9)^2} = \sqrt{6^2 + (-12)^2} = \sqrt{36 + 144} = \sqrt{180} = 6\sqrt{5} \)

Midpoint: \( \left( \frac{0 - 6}{2}, \frac{-3 + 9}{2} \right) = (-3, 3) \)

119. \(-6(2 - x) - 12 = 36 \)  
\(-12 + 6x - 12 = 36 \)  
\(-24 + 6x = 36 \)  
\(6x = 60 \)  
\(x = 10 \)

120. \(4(x + 7) - 9 = -6(-x - 1) \)  
\(4x + 28 - 9 = 6x + 6 \)  
\(-2x = -13 \)  
\(x = \frac{13}{2} \)

121. \(14x^2 + 5x - 1 = 0 \)

\((7x - 1)(2x + 1) = 0 \)

\(7x - 1 = 0 \) or \(2x + 1 = 0 \)

\(7x = 1 \) or \(2x = -1 \)

\(x = \frac{1}{7} \) or \(x = -\frac{1}{2} \)

122. \(x^3 + 5x^2 - 4x - 20 = 0 \)

\(x^2(x + 5) - 4(x + 5) = 0 \)

\((x + 5)(x^2 - 4) = 0 \)

\(x + 5 = 0 \Rightarrow x = -5 \)

\(x^2 - 4 = 0 \Rightarrow x = \pm 2 \)

123. \((-3, 10) \)

124. If \( y > 0 \), then the point \((x, y)\) could be located in either Quadrant I or II.

125. Answers will vary.

---

Section 1.8 Other Types of Inequalities

- You should be able to solve inequalities.
  - Find the critical number.
    - Values that make the expression zero
    - Values that make the expression undefined
  - Test one value in each test interval on the real number line resulting from the critical numbers.
  - Determine the solution intervals.

**Vocabulary Check**

1. critical; test intervals
2. zeroes; undefined values
3. \( P = R - C \)
1. $x^2 - 3 < 0$
   (a) $x = 3$  
   (b) $x = 0$  
   (c) $x = \frac{3}{2}$  
   (d) $x = -5$
   
   $(3)^2 - 3 < 0$  
   $(0)^2 - 3 < 0$  
   $(\frac{3}{2})^2 - 3 < 0$  
   $(-5)^2 - 3 < 0$

   6 $\not< 0$  
   $-3 < 0$  
   $-\frac{3}{2} < 0$  
   $22 < 0$

   No, $x = 3$ is not a solution.  
   Yes, $x = 0$ is a solution.  
   Yes, $x = \frac{3}{2}$ is a solution.  
   No, $x = -5$ is not a solution.

2. $x^2 - x - 12 \geq 0$
   (a) $x = 5$  
   (b) $x = 0$  
   (c) $x = -4$  
   (d) $x = -3$

   $(5)^2 - (5) - 12 \geq 0$  
   $(0)^2 - 0 - 12 \geq 0$  
   $(-4)^2 - (-4) - 12 \geq 0$  
   $(-3)^2 - (-3) - 12 \geq 0$

   $8 \geq 0$  
   $-12 \not\geq 0$  
   $16 + 4 - 12 \geq 0$  
   $9 + 3 - 12 \geq 0$

   Yes, $x = 5$ is a solution.  
   No, $x = 0$ is not a solution.  
   Yes, $x = -4$ is a solution.  
   Yes, $x = -3$ is a solution.

3. $\frac{x + 2}{x - 4} \geq 3$
   (a) $x = 5$  
   (b) $x = 4$  
   (c) $x = -\frac{9}{2}$  
   (d) $x = \frac{9}{2}$

   $\frac{5 + 2}{5 - 4} \geq 3$  
   $\frac{4 + 2}{4 - 4} \geq 3$  
   $\frac{-\frac{9}{2} + 2}{-\frac{9}{2} - 4} \geq 3$  
   $\frac{\frac{9}{2} + 2}{\frac{9}{2} - 4} \geq 3$

   $7 \geq 3$  
   $6 \not= 0$ is undefined.  
   $5 \not= 3$  
   $13 \geq 3$

   Yes, $x = 5$ is a solution.  
   No, $x = 4$ is not a solution.  
   No, $x = -\frac{9}{2}$ is not a solution.  
   Yes, $x = \frac{9}{2}$ is a solution.

4. $\frac{3x^2}{x^2 + 4} < 1$
   (a) $x = -2$  
   (b) $x = -1$  
   (c) $x = 0$  
   (d) $x = 3$

   $\frac{3(-2)^2}{(-2)^2 + 4} < 1$  
   $\frac{3(-1)^2}{(-1)^2 + 4} < 1$  
   $\frac{3(0)^2}{(0)^2 + 4} < 1$  
   $\frac{3(3)^2}{(3)^2 + 4} < 1$

   $\frac{12}{8} \not< 1$  
   $\frac{3}{5} \not< 1$  
   $0 < 1$  
   $\frac{27}{13} \not< 1$

   No, $x = -2$ is not a solution.  
   Yes, $x = -1$ is a solution.  
   Yes, $x = 0$ is a solution.  
   No, $x = 3$ is not a solution.

5. $2x^2 - x - 6 = (2x + 3)(x - 2)$

   $2x + 3 = 0 \implies x = -\frac{3}{2}$

   $x - 2 = 0 \implies x = 2$

   Critical numbers: $x = -\frac{3}{2}, x = 2$

6. $9x^3 - 25x^2 = 0$

   $x^2(9x - 25) = 0$

   $x = 0 \implies x = 0$

   $9x - 25 = 0 \implies x = \frac{25}{9}$

   The critical numbers are 0 and $\frac{25}{9}$.

7. $2 + \frac{3}{x - 5} = \frac{2(x - 5) + 3}{x - 5}$

   $2x - 7 = 0 \implies x = \frac{7}{2}$

   $x - 5 = 0 \implies x = 5$

   Critical numbers: $x = \frac{7}{2}, x = 5$
### Section 1.8 Other Types of Inequalities

#### Solution set:

<table>
<thead>
<tr>
<th>Interval</th>
<th>x-Value</th>
<th>Value of ( x^2 - 9 )</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -3))</td>
<td>(x = -4)</td>
<td>(16 - 9 = 7)</td>
<td>Positive</td>
</tr>
<tr>
<td>((-3, 3))</td>
<td>(x = 0)</td>
<td>(0 - 9 = -9)</td>
<td>Negative</td>
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<td>((3, \infty))</td>
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Solution set: \([-3, 3]\)

#### Solution intervals:

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Solution set: \([-3, 3]\)

#### Critical numbers: \(x = \pm 3\)

**Test intervals:** \((-\infty, -3), (-3, 3), (3, \infty)\)

**Test:** Is \((x + 3)(x - 3) \leq 0\)?

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<td>(16 - 9 = 7)</td>
<td>Positive</td>
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Solution set: \([-3, 3]\)
13. \( x^2 + 4x + 4 \geq 9 \)
\( x^2 + 4x - 5 \geq 0 \)
\( (x + 5)(x - 1) \geq 0 \)
Critical intervals: \( x = -5, x = 1 \)
Test: Is \( (x + 5)(x - 1) \geq 0 \)?
Interval | x-Value | Value of \( (x + 5)(x - 1) \) | Conclusion
--- | --- | --- | ---
\( (-\infty, -5) \) | \( x = -6 \) | \( (-1)(-7) = 7 \) | Negative
\( (-5, 1) \) | \( x = 0 \) | \( (5)(-1) = -5 \) | Negative
\( (1, \infty) \) | \( x = 2 \) | \( (7)(1) = 7 \) | Positive
Solution set: \( (-\infty, -5] \cup [1, \infty) \)

15. \( x^2 + x < 6 \)
\( x^2 + x - 6 < 0 \)
\( (x + 3)(x - 2) < 0 \)
Critical intervals: \( x = -3, x = 2 \)
Test: Is \( (x + 3)(x - 2) < 0 \)?
Interval | x-Value | Value of \( (x + 3)(x - 2) \) | Conclusion
--- | --- | --- | ---
\( (-\infty, -3) \) | \( x = -4 \) | \( (-1)(-6) = 6 \) | Negative
\( (-3, 2) \) | \( x = 0 \) | \( (3)(-2) = -6 \) | Negative
\( (2, \infty) \) | \( x = 3 \) | \( (6)(1) = 6 \) | Positive
Solution set: \( (-3, 2) \)

16. \( x^2 + 2x > 3 \)
\( x^2 + 2x - 3 > 0 \)
\( (x + 3)(x - 1) > 0 \)
Critical intervals: \( x = -3, x = 1 \)
Test: Is \( (x + 3)(x - 1) > 0 \)?
Interval | x-Value | Value of \( (x + 3)(x - 1) \) | Conclusion
--- | --- | --- | ---
\( (-\infty, -3) \) | \( x = -4 \) | \( (-1)(-6) = 6 \) | Positive
\( (-3, 2) \) | \( x = 0 \) | \( (3)(-2) = -6 \) | Negative
\( (2, \infty) \) | \( x = 3 \) | \( (6)(1) = 6 \) | Positive
Solution intervals: \( (-\infty, -3) \cup (1, \infty) \)

18. \( x^2 - 4x - 1 > 0 \)
\( x = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5} \)
Critical intervals: \( x = 2 - \sqrt{5}, x = 2 + \sqrt{5} \)
Test: Is \( x^2 - 4x - 1 > 0 \)?
Interval | x-Value | Value of \( x^2 - 4x - 1 \) | Conclusion
--- | --- | --- | ---
\( (-\infty, 2 - \sqrt{5}) \) | \( x = 2 - \sqrt{5} \) | \( 2 - 4(2 - \sqrt{5}) = \sqrt{5} - 4 \) | Positive
\( (2 - \sqrt{5}, 2 + \sqrt{5}) \) | \( x = 2 + \sqrt{5} \) | \( 2 + 4(2 + \sqrt{5}) = \sqrt{5} + 4 \) | Positive
\( (2 + \sqrt{5}, \infty) \) | \( x = 2 + \sqrt{5} \) | \( 2 - 4x - 1 = 4\sqrt{5} - 3 \) | Positive
Solution intervals: \( (-\infty, 2 - \sqrt{5}) \cup (2 + \sqrt{5}, \infty) \)
19. \( x^2 + 8x - 5 \geq 0 \)
\[
x^2 + 8x - 5 = 0
\]
Complete the square.
\[
x^2 + 8x + 16 = 5 + 16
\]
\[
(x + 4)^2 = 21
\]
\[
x + 4 = \pm \sqrt{21}
\]
\[
x = -4 \pm \sqrt{21}
\]
Critical numbers: \( x = -4 \pm \sqrt{21} \)
Test intervals: \( (-\infty, -4 - \sqrt{21}), (-4 - \sqrt{21}, -4 + \sqrt{21}), (-4 + \sqrt{21}, \infty) \)
Test: Is \( x^2 + 8x - 5 \geq 0 \)?

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</tr>
</thead>
<tbody>
<tr>
<td>( (-\infty, -4 - \sqrt{21}) )</td>
<td>( x = -10 )</td>
<td>( 100 - 80 - 5 = 15 )</td>
<td>Positive</td>
</tr>
<tr>
<td>( (-4 - \sqrt{21}, -4 + \sqrt{21}) )</td>
<td>( x = 0 )</td>
<td>( 0 + 0 - 5 = -5 )</td>
<td>Negative</td>
</tr>
<tr>
<td>( (-4 + \sqrt{21}, \infty) )</td>
<td>( x = 2 )</td>
<td>( 4 + 16 - 5 = 15 )</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Solution set: \( (-\infty < -4 - \sqrt{21}] \cup [-4 + \sqrt{21}, \infty) \).

20. \( -2x^2 + 6x + 15 \leq 0 \)
\[
2x^2 - 6x - 15 \geq 0
\]
\[
x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)} = \frac{6 \pm \sqrt{36 + 120}}{4}
\]
\[
= \frac{6 \pm 2\sqrt{39}}{4} = \frac{3 \pm \sqrt{39}}{2}
\]
Critical numbers: \( x = \frac{3}{2} - \frac{\sqrt{39}}{2}, x = \frac{3}{2} + \frac{\sqrt{39}}{2} \)
Test intervals:
\[
\left( -\infty, \frac{3}{2} - \frac{\sqrt{39}}{2} \right) \Rightarrow -2x^2 + 6x + 15 < 0
\]
\[
\left( \frac{3}{2} - \frac{\sqrt{39}}{2}, \frac{3}{2} + \frac{\sqrt{39}}{2} \right) \Rightarrow -2x^2 + 6x + 15 > 0
\]
\[
\left( \frac{3}{2} + \frac{\sqrt{39}}{2}, \infty \right) \Rightarrow -2x^2 + 6x + 15 < 0
\]
Solution interval: \( \left( -\infty, \frac{3}{2} - \frac{\sqrt{39}}{2} \right] \cup \left[ \frac{3}{2} + \frac{\sqrt{39}}{2}, \infty \right) \)

21. \( x^3 - 3x^2 - x + 3 > 0 \)
\[
x^2(x - 3) - 1(x - 3) > 0
\]
\[
(x^2 - 1)(x - 3) > 0
\]
\[
(x + 1)(x - 1)(x - 3) > 0
\]
Critical numbers: \( x = \pm 1, x = 3 \)
Test intervals: \( (-\infty, -1), (-1, 1), (1, 3), (3, \infty) \)
Test: Is \( (x + 1)(x - 1)(x - 3) > 0 \)?

<table>
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<tr>
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<th>Value of ( (x + 1)(x - 1)(x - 3) )</th>
<th>Conclusion</th>
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<tbody>
<tr>
<td>( (-\infty, -1) )</td>
<td>( x = -2 )</td>
<td>( -1(-3)(-5) = -15 )</td>
<td>Negative</td>
</tr>
<tr>
<td>( (-1, 1) )</td>
<td>( x = 0 )</td>
<td>( (1)(-1)(-3) = 3 )</td>
<td>Positive</td>
</tr>
<tr>
<td>( (1, 3) )</td>
<td>( x = 2 )</td>
<td>( 3(1)(-1) = -3 )</td>
<td>Negative</td>
</tr>
<tr>
<td>( (3, \infty) )</td>
<td>( x = 4 )</td>
<td>( 5(3)(1) = 15 )</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Solution set: \( (-1, 1) \cup (3, \infty) \)

22. \( x^3 + 2x^2 - 4x - 8 \leq 0 \)
\[
x^3(x + 2) - 4(x + 2) \leq 0
\]
\[
(x + 2)(x^2 - 4) \leq 0
\]
Critical numbers: \( x = -2, x = 2 \)
Test intervals: \( (-\infty, -2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0 \)
\[
(-2, 2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0
\]
\[
(2, \infty) \Rightarrow x^3 + 2x^2 - 4x - 8 > 0
\]
Solution interval: \( (-\infty, 2] \)
23. \[ x^3 - 2x^2 - 9x - 2 \geq -20 \]
\[ x^3 - 2x^2 - 9x + 18 \geq 0 \]
\[ x^2(x - 2) - 9(x - 2) \geq 0 \]
\[ (x - 2)(x^2 - 9) \geq 0 \]
\[ (x - 2)(x + 3)(x - 3) \geq 0 \]
Critical numbers: \( x = 2, x = \pm 3 \)

Test intervals: \((-\infty, -3), (-3, 2), (2, 3), (3, \infty)\)

<table>
<thead>
<tr>
<th>Interval</th>
<th>( x )-Value</th>
<th>Value of ((x - 2)(x + 3)(x - 3))</th>
<th>Conclusion</th>
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<tbody>
<tr>
<td>((-\infty, -3))</td>
<td>( x = -4 )</td>
<td>((-6)(-1)(-7) = -42)</td>
<td>Negative</td>
</tr>
<tr>
<td>((-3, 2))</td>
<td>( x = 0 )</td>
<td>((-2)(3)(-3) = 18)</td>
<td>Positive</td>
</tr>
<tr>
<td>((2, 3))</td>
<td>( x = 2.5 )</td>
<td>((0.5)(5.5)(-0.5) = -1.375)</td>
<td>Negative</td>
</tr>
<tr>
<td>((3, \infty))</td>
<td>( x = 4 )</td>
<td>((2)(7)(1) = 14)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Solution set: \([-3, 2] \cup [3, \infty)\)

24. \[ 2x^3 + 13x^2 - 8x - 46 \geq 6 \]
\[ 2x^3 + 13x^2 - 8x - 52 \geq 0 \]
\[ x^2(2x + 13) - 4(2x + 13) \geq 0 \]
\[ (2x + 13)(x^2 - 4) \geq 0 \]

Critical numbers: \( x = -\frac{13}{2}, x = -2, x = 2 \)

Test intervals: \((-\infty, -\frac{13}{2}), \left(-\frac{13}{2}, -2\right), \left(-2, \frac{13}{2}\right), \left(\frac{13}{2}, \infty\right)\)

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<th>Interval</th>
<th>( x )-Value</th>
<th>Value of ((2x + 13)(x^2 - 4))</th>
<th>Conclusion</th>
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</thead>
<tbody>
<tr>
<td>((-\infty, -\frac{13}{2}))</td>
<td>( x = -7 )</td>
<td></td>
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</tr>
<tr>
<td>((-\frac{13}{2}, -2))</td>
<td>( x = -4 )</td>
<td>((1)(-8) = -8)</td>
<td></td>
</tr>
<tr>
<td>((-2, \frac{13}{2}))</td>
<td>( x = 0 )</td>
<td>((13)(-9) = -117)</td>
<td></td>
</tr>
<tr>
<td>(\left(\frac{13}{2}, \infty\right))</td>
<td>( x = 6 )</td>
<td>((13)(25) = 325)</td>
<td></td>
</tr>
</tbody>
</table>

Solution interval: \([-\frac{13}{2}, -2] \cup [2, \infty)\)

25. \[ 4x^2 - 4x + 1 \leq 0 \]
\[ (2x - 1)^2 \leq 0 \]

Critical number: \( x = \frac{1}{2} \)

Test intervals: \((-\infty, \frac{1}{2}), (\frac{1}{2}, \infty)\)

<table>
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<td>(\left(\frac{1}{2}, \infty\right))</td>
<td>( x = 1 )</td>
<td>((1)^2 = 1)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Solution set: \( x = \frac{1}{2} \)

26. \[ x^2 + 3x + 8 > 0 \]

The critical numbers are imaginary:
\[ x = -\frac{3}{2} \pm \frac{\sqrt{23}}{2} \]

So the set of real numbers is the solution set.
27. \(4x^3 - 6x^2 < 0\)
\(2x^2(2x - 3) < 0\)
Critical numbers: \(x = 0, x = \frac{3}{2}\)
Test intervals: \((-\infty, 0), (0, \frac{3}{2}), (\frac{3}{2}, \infty)\)
Test: Is \(2x^2(2x - 3) < 0\)?
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-\infty, 0) \cup (0, \frac{3}{2})\)

28. \(4x^3 - 12x^2 > 0\)
\(4x^2(x - 3) > 0\)
Critical numbers: \(x = 0, x = 3\)
Test intervals: \((-\infty, 0) \Rightarrow 4x^2(x - 3) < 0\)
\((0, 3) \Rightarrow 4x^2(x - 3) < 0\)
\((3, \infty) \Rightarrow 4x^2(x - 3) > 0\)
Solution interval: \((3, \infty)\)

29. \(x^3 - 4x \geq 0\)
\(x(x + 2)(x - 2) \geq 0\)
Critical numbers: \(x = 0, x = \pm 2\)
Test intervals: \((-\infty, -2), (-2, 0), (0, 2), (2, \infty)\)
Test: Is \(x(x + 2)(x - 2) \geq 0\)?
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \([-2, 0] \cup [2, \infty)\)

30. \(2x^3 - x^4 \leq 0\)
\(x^3(2 - x) \leq 0\)
Critical numbers: \(x = 0, x = 2\)
Test intervals: \((-\infty, 0) \Rightarrow x^3(2 - x) < 0\)
\((0, 2) \Rightarrow x^3(2 - x) > 0\)
\((2, \infty) \Rightarrow x^3(2 - x) < 0\)
Solution intervals: \((-\infty, 0] \cup [2, \infty)\)

31. \((x - 1)^2(x + 2)^3 \geq 0\)
Critical numbers: \(x = 1, x = -2\)
Test intervals: \((-\infty, -2), (-2, 1), (1, \infty)\)
Test: Is \((x - 1)^2(x + 2)^3 \geq 0\)?
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \([-2, \infty)\)

32. \(x^4(x - 3) \leq 0\)
Critical numbers: \(x = 0, x = 3\)
Test intervals: \((-\infty, 0) \Rightarrow x^4(x - 3) < 0\)
\((0, 3) \Rightarrow x^4(x - 3) < 0\)
\((3, \infty) \Rightarrow x^4(x - 3) > 0\)
Solution intervals: \((-\infty, 0] \cup [0, 3] \cup (-\infty, 3]\)

33. \(y = -x^2 + 2x + 3\)
(a) \(y \leq 0\) when \(x \leq -1\) or \(x \geq 3\).
(b) \(y \geq 3\) when \(0 \leq x \leq 2\).

34. \(y = \frac{1}{2}x^2 - 2x + 1\)
(a) \(y \leq 0\)
\[\frac{1}{2}x^2 - 2x + 1 \leq 0\]
\[x^2 - 4x + 2 \leq 0\]
\[x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}\]
\(y \leq 0\) when \(2 - \sqrt{2} \leq x \leq 2 + \sqrt{2}\).
(b) \(y \geq 7\)
\[\frac{1}{2}x^2 - 2x + 1 \geq 7\]
\[x^2 - 4x - 12 \geq 0\]
\[(x - 6)(x + 2) \geq 0\]
\(x \leq -2\) or \(x \geq 6\).
\(y \geq 7\) when \(x \leq -2\) or \(x \geq 6\).
35. \( y = \frac{1}{2}x^3 - \frac{1}{2}x \)  
(a) \( y \geq 0 \) when \(-2 \leq x \leq 0, 2 \leq x < \infty \).  
(b) \( y \leq 6 \) when \( x \leq 4 \).

36. \( y = x^3 - x^2 - 16x + 16 \)  
(a) \( y \leq 0 \)  
\[ x^3 - x^2 - 16x + 16 \leq 0 \]  
\[ x^2(x - 1) - 16(x - 1) \leq 0 \]  
\[ (x - 1)(x^2 - 16) \leq 0 \]  
\( y \leq 0 \) when \(-\infty < x \leq -4, 1 \leq x \leq 4 \).

(b) \( y \geq 36 \)  
\[ x^3 - x^2 - 16x + 16 \geq 36 \]  
\[ x^2(x - 1) - 16(x - 1) \geq 0 \]  
\[ (x + 2)(x - 5)(x + 2) \geq 0 \]  
\( y \geq 36 \) when \( x = -2, 5 \leq x < \infty \).

37. \( \frac{1}{x} - x > 0 \)

\[ \frac{1 - x^2}{x} > 0 \]

Critical numbers: \( x = 0, x = \pm 1 \)

Test intervals: \((-\infty, -1), (-1, 0), (0, 1), (1, \infty)\)

Test: Is \( \frac{1 - x^2}{x} > 0 \)?

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \((-\infty, -1) \cup (0, 1)\)

38. \( \frac{1}{x} - 4 < 0 \)

\[ \frac{1 - 4x}{x} < 0 \]

Critical numbers: \( x = 0, x = \frac{1}{4} \)

Test intervals: \((-\infty, 0) \Rightarrow \frac{1 - 4x}{x} < 0\)

\[ \left(0, \frac{1}{4}\right) \Rightarrow \frac{1 - 4x}{x} > 0 \]

\[ \left(\frac{1}{4}, \infty\right) \Rightarrow \frac{1 - 4x}{x} < 0 \]

Solution interval: \((-\infty, 0) \cup \left(\frac{1}{4}, \infty\right)\)

39. \( \frac{x + 6}{x + 1} - 2 < 0 \)

\[ \frac{x + 6 - 2(x + 1)}{x + 1} < 0 \]

\[ \frac{4 - x}{x + 1} < 0 \]

Critical numbers: \( x = -1, x = 4 \)

Test intervals: \((-\infty, -1), (-1, 4), (4, \infty)\)

Test: Is \( \frac{4 - x}{x + 1} < 0 \)?

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \((-\infty, -1) \cup (4, \infty)\)

40. \( \frac{x + 12}{x + 2} - 3 \geq 0 \)

\[ \frac{x + 12 - 3(x + 2)}{x + 2} \geq 0 \]

\[ \frac{6 - 2x}{x + 2} \geq 0 \]

Critical numbers: \( x = -2, x = 3 \)

Test intervals: \((-\infty, -2) \Rightarrow \frac{6 - 2x}{x + 2} < 0\)

\[ (-2, 3) \Rightarrow \frac{6 - 2x}{x + 2} > 0 \]

\[ (3, \infty) \Rightarrow \frac{6 - 2x}{x + 2} < 0 \]

Solution interval: \((-2, 3)\)
41. \[
\frac{3x - 5}{x - 5} > 4
\]
\[
\frac{3x - 5}{x - 5} - 4 > 0
\]
\[
\frac{3x - 5 - 4(x - 5)}{x - 5} > 0
\]
\[
\frac{15 - x}{x - 5} > 0
\]
Critical numbers: \(x = 5, x = 15\)
Test intervals: \((-\infty, 5), (5, 15), (15, \infty)\)
Test: \(\frac{15 - x}{x - 5} > 0\)?
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((5, 15)\)

42. \[
\frac{5 + 7x}{1 + 2x} < 4
\]
\[
\frac{5 + 7x - 4(1 + 2x)}{1 + 2x} < 0
\]
\[
\frac{1 - x}{1 + 2x} < 0
\]
Critical numbers: \(x = -\frac{1}{2}, x = 1\)
Test intervals: \((-\infty, -\frac{1}{2}), (-\frac{1}{2}, 1), (1, \infty)\)
Test: \(\frac{1 - x}{1 + 2x} > 0\)
Solution intervals: \((-\infty, -\frac{1}{2}) \cup (1, \infty)\)

43. \[
\frac{4}{x + 5} > \frac{1}{2x + 3}
\]
\[
\frac{4}{x + 5} - \frac{1}{2x + 3} > 0
\]
\[
\frac{4(2x + 3) - (x + 5)}{(x + 5)(2x + 3)} > 0
\]
\[
\frac{7x + 7}{(x + 5)(2x + 3)} > 0
\]
Critical numbers: \(x = -1, x = -5, x = -\frac{3}{2}\)
Test intervals: \((-\infty, -5), \left(-5, -\frac{3}{2}\right), \left(-\frac{3}{2}, -1\right), (-1, \infty)\)
Test: \(\frac{7(x + 1)}{(x + 5)(2x + 3)} > 0\)?
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-5, -\frac{3}{2}) \cup (-1, \infty)\)

44. \[
\frac{5}{x - 6} > \frac{3}{x + 2}
\]
\[
\frac{5(x + 2) - 3(x - 6)}{(x - 6)(x + 2)} > 0
\]
\[
\frac{2x + 28}{(x - 6)(x + 2)} > 0
\]
Critical numbers: \(x = -14, x = -2, x = 6\)
Test intervals: \((-\infty, -14), (-14, -2), (-2, 6), (6, \infty)\)
Test: \(\frac{2x + 28}{(x - 6)(x + 2)} < 0\)
Solution intervals: \((-14, -2) \cup (6, \infty)\)
45. \[
\frac{1}{x - 3} \leq \frac{9}{4x + 3}
\]
\[
\frac{1}{x - 3} - \frac{9}{4x + 3} \leq 0
\]
\[
\frac{4x + 3 - 9(x - 3)}{(x - 3)(4x + 3)} \leq 0
\]
\[
\frac{30 - 5x}{(x - 3)(4x + 3)} \leq 0
\]
Critical numbers: \(x = 3, x = -\frac{3}{4}, x = 6\)
Test intervals: \((-\infty, -\frac{3}{4}), (-\frac{3}{4}, 3), (3, 6), (6, \infty)\)
Test: Is \(\frac{30 - 5x}{(x - 3)(4x + 3)} \leq 0?\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-\frac{3}{2}, 3) \cup (6, \infty)\)

46. \[
\frac{1}{x} \geq \frac{1}{x + 3}
\]
\[
\frac{1(x + 3) - 1(x)}{x(x + 3)} \geq 0
\]
\[
\frac{3}{x(x + 3)} \geq 0
\]
Critical numbers: \(x = -3, x = 0\)
Test intervals: \((-\infty, -3) \Rightarrow \frac{3}{x(x + 3)} > 0\)
\((-3, 0) \Rightarrow \frac{3}{x(x + 3)} < 0\)
\((0, \infty) \Rightarrow \frac{3}{x(x + 3)} > 0\)
Solution intervals: \((-\infty, -3) \cup (0, \infty)\)

47. \[
\frac{x^2 + 2x}{x^2 - 9} \leq 0
\]
\[
\frac{x(x + 2)}{(x + 3)(x - 3)} \leq 0
\]
Critical numbers: \(x = 0, x = -2, x = \pm 3\)
Test intervals: \((-\infty, -3), (-3, -2), (-2, 0), (0, 3), (3, \infty)\)
Test: Is \(\frac{x(x + 2)}{(x + 3)(x - 3)} \leq 0?\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-3, -2) \cup (0, 3)\)

48. \[
\frac{x^2 + x - 6}{x} \geq 0
\]
\[
\frac{(x + 3)(x - 2)}{x} \geq 0
\]
Critical numbers: \(x = -3, x = 0, x = 2\)
Test intervals: \((-\infty, -3) \Rightarrow \frac{(x + 3)(x - 2)}{x} < 0\)
\((-3, 0) \Rightarrow \frac{(x + 3)(x - 2)}{x} > 0\)
\((0, 2) \Rightarrow \frac{(x + 3)(x - 2)}{x} < 0\)
\((2, \infty) \Rightarrow \frac{(x + 3)(x - 2)}{x} > 0\)
Solution intervals: \([-3, 0) \cup [2, \infty)\)
49. \[ \frac{5}{x - 1} - \frac{2x}{x + 1} < 1 \]

\[ \frac{5}{x - 1} - \frac{2x}{x + 1} - 1 < 0 \]

\[ \frac{5(x + 1) - 2x(x - 1) - (x - 1)(x + 1)}{(x - 1)(x + 1)} < 0 \]

\[ \frac{5x + 5 - 2x^2 + 2x - x^2 + 1}{(x - 1)(x + 1)} < 0 \]

\[ \frac{-3x^2 + 7x + 6}{(x - 1)(x + 1)} < 0 \]

\[ \frac{-(3x + 2)(x - 3)}{(x - 1)(x + 1)} < 0 \]

Critical numbers: \( x = -\frac{2}{3}, x = 3, x = \pm 1 \)

Test intervals: \( (-\infty, -1), (-1, -\frac{2}{3}), (-\frac{2}{3}, 1), (1, 3), (3, \infty) \)

Test: Is \( \frac{-(3x + 2)(x - 3)}{(x - 1)(x + 1)} < 0? \)

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \( (-\infty, -1) \cup \left(-\frac{2}{3}, 1\right) \cup (3, \infty) \)

50. \[ \frac{3x}{x - 1} \leq \frac{x}{x + 4} + 3 \]

\[ \frac{3x(x + 4) - x(x - 1) - 3(x + 4)(x - 1)}{(x - 1)(x + 4)} \leq 0 \]

\[ \frac{-x^2 + 4x + 12}{(x - 1)(x + 4)} \leq 0 \]

\[ \frac{-x - 6)(x + 2)}{(x - 1)(x + 4)} \leq 0 \]

Critical numbers: \( x = -4, x = -2, x = 1, x = 6 \)

Test intervals: \( (-\infty, -4) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0 \)

\( (-4, -2) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} > 0 \)

\( (-2, 1) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0 \)

\( (1, 6) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} > 0 \)

\( (6, \infty) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0 \)

Solution intervals: \( (-\infty, -4), [-2, 1], [6, \infty) \)

51. \( y = \frac{3x}{x - 2} \)

(a) \( y \leq 0 \) when \( 0 \leq x < 2 \).

(b) \( y \geq 6 \) when \( 2 < x \leq 4 \).

52. \( y = \frac{2(x - 2)}{x + 1} \)

(a) \( y \leq 0 \)

\[ \frac{2(x - 2)}{x + 1} \leq 0 \]

\( y \leq 0 \) when \( -1 < x \leq 2 \).

(b) \( y \geq 8 \)

\[ \frac{2(x - 2)}{x + 1} \geq 8 \]

\[ \frac{2(x - 2) - 8(x + 1)}{x + 1} \geq 0 \]

\[ \frac{-6x - 12}{x + 1} \geq 0 \]

\[ \frac{-6(x + 2)}{x + 1} \geq 0 \]

\( y \geq 8 \) when \( -2 \leq x < -1 \).
53. \( y = \frac{2x^2}{x^2 + 4} \)

(a) \( y \geq 1 \) when \( x \leq -2 \) or \( x \geq 2 \).

This can also be expressed as \(|x| \geq 2\).

(b) \( y \leq 2 \) for all real numbers \( x \).

This can also be expressed as \(-\infty < x < \infty\).

54. \( y = \frac{5x}{x^2 + 4} \)

(a) \( y \geq 1 \)

\( \frac{5x}{x^2 + 4} \geq 1 \)

\( \frac{5x - (x^2 + 4)}{x^2 + 4} \geq 0 \)

\( -(x - 4)(x - 1) \geq 0 \)

\( y \geq 1 \) when \( 1 \leq x \leq 4 \).

(b) \( y \leq 0 \)

\( \frac{5x}{x^2 + 4} \leq 0 \)

\( y \leq 0 \) when \(-\infty < x \leq 0\).

55. \( 4 - x^2 \geq 0 \)

\((2 + x)(2 - x) \geq 0\)

Critical numbers: \( x = \pm 2\)

Test intervals: \((-\infty, -2), (-2, 2), (2, \infty)\)

Test: Is \( 4 - x^2 \geq 0 \)?

By testing an \( x \)-value in each test interval in the inequality, we see that the domain set is: \([ -2, 2 ]\)

56. \( x^2 - 4 \geq 0 \)

\((x + 2)(x - 2) \geq 0\)

Critical numbers: \( x = -2, x = 2\)

Test intervals: \((-\infty, -2) \Rightarrow (x + 2)(x - 2) > 0 \)

\((-2, 2) \Rightarrow (x + 2)(x - 2) < 0 \)

\((2, \infty) \Rightarrow (x + 2)(x - 2) > 0\)

Domain: \((-\infty, -2] \cup [2, \infty)\)

57. \( x^2 - 7x + 12 \geq 0 \)

\((x - 3)(x - 4) \geq 0\)

Critical numbers: \( x = 3, x = 4\)

Test intervals: \((-\infty, 3), (3, 4), (4, \infty)\)

Test: Is \((x - 3)(x - 4) \geq 0\)?

By testing an \( x \)-value in each test interval in the inequality, we see that the domain set is: \((-\infty, 3] \cup [4, \infty)\)

58. \( 144 - 9x^2 \geq 0 \)

\(9(4 - x)(4 + x) \geq 0\)

Critical numbers: \( x = -4, x = 4\)

Test intervals: \((-\infty, -4) \Rightarrow 9(4 - x)(4 + x) < 0\)

\((-4, 4) \Rightarrow 9(4 - x)(4 + x) > 0\)

\((4, \infty) \Rightarrow 9(4 - x)(4 + x) < 0\)

Domain: \([-4, 4]\)

59. \( \frac{x}{x^2 - 2x - 35} \geq 0 \)

\( \frac{x}{(x + 5)(x - 7)} \geq 0\)

Critical numbers: \( x = 0, x = -5, x = 7\)

Test intervals: \((-\infty, -5), (-5, 0), (0, 7), (7, \infty)\)

Test: Is \( \frac{x}{(x + 5)(x - 7)} \geq 0\)?

By testing an \( x \)-value in each test interval in the inequality, we see that the domain set is: \((-5, 0] \cup (7, \infty)\)
60. \( \frac{x}{x^2 - 9} \geq 0 \)
\( \frac{x}{(x + 3)(x - 3)} \geq 0 \)
Critical numbers: \( x = -3, x = 0, x = 3 \)
Test intervals: \((-\infty, -3) \Rightarrow \frac{x}{(x + 3)(x - 3)} < 0\)
\((-3, 0) \Rightarrow \frac{x}{(x + 3)(x - 3)} > 0\)
\((0, 3) \Rightarrow \frac{x}{(x + 3)(x - 3)} < 0\)
\((3, \infty) \Rightarrow \frac{x}{(x + 3)(x - 3)} > 0\)
Domain: \((-3, 0] \cup (3, \infty)\)

62. \(-1.3x^2 + 3.78 \geq 2.12\)
\(-1.3x^2 + 1.66 > 0\)
Critical numbers: ±1.13
Test intervals: \((-\infty, -1.13), (-1.13, 1.13), (1.13, \infty)\)
Solution set: \((-1.13, 1.13)\)

64. \(1.2x^2 + 4.8x + 3.1 < 5.3\)
\(1.2x^2 + 4.8x - 2.2 < 0\)
Critical numbers: -4.42, 0.42
Test intervals: \((-\infty, -4.42), (-4.42, 0.42), (0.42, \infty)\)
Solution set: \((-4.42, 0.42)\)

66. \(\frac{2}{3.1x - 3.7} > 5.8\)
\(\frac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0\)
\(\frac{23.46 - 17.98x}{3.1x - 3.7} > 0\)
Critical numbers: \(x = 1.19, x = 1.30\)
Test intervals: \((-\infty, 1.19) \Rightarrow \frac{23.46 - 17.98x}{3.1x - 3.7} < 0\)
\((1.19, 1.30) \Rightarrow \frac{23.46 - 17.98x}{3.1x - 3.7} > 0\)
\((1.30, \infty) \Rightarrow \frac{23.46 - 17.98x}{3.1x - 3.7} < 0\)
Solution interval: \((1.19, 1.30)\)

61. \(0.4x^2 + 5.26 < 10.2\)
\(0.4x^2 - 4.94 < 0\)
\(0.4(x^2 - 12.35) < 0\)
Critical numbers: \(x \approx \pm 3.51\)
Test intervals: \((-\infty, -3.51), (-3.51, 3.51), (3.51, \infty)\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-3.51, 3.51)\)

63. \(-0.5x^2 + 12.5x + 1.6 > 0\)
The zeros are \(x = \frac{-12.5 \pm \sqrt{(12.5)^2 - 4(-0.5)(1.6)}}{2(-0.5)}\).
Critical numbers: \(x \approx -0.13, x \approx 25.13\)
Test intervals: \((-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-0.13, 25.13)\)

65. \(\frac{1}{2.3x - 5.2} > 3.4\)
\(\frac{1}{2.3x - 5.2} - 3.4 > 0\)
\(\frac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0\)
\(-7.82x + 18.68 > 0\)
\(2.3x - 5.2 > 0\)
Critical numbers: \(x \approx 2.39, x \approx 2.26\)
Test intervals: \((-\infty, 2.26), (2.26, 2.39), (2.39, \infty)\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((2.26, 2.39)\)
67. \( s = -16t^2 + v_0t + s_0 = -16t^2 + 160t \)
   (a) \(-16t^2 + 160t = 0\)
       \[-16(t - 10) = 0\]
       \(t = 0, t = 10\)
   It will be back on the ground in 10 seconds.
   (b) \(-16t^2 + 160t > 384\)
       \(-16t^2 + 160t - 384 > 0\)
       \(-16(t^2 - 10t + 24) > 0\)
       \(t^2 - 10t + 24 < 0\)
       \((t - 4)(t - 6) < 0\)
       \(4 < t < 6\) seconds

68. \( s = -16t^2 + v_0t + s_0 = -16t^2 + 128t \)
   (a) \(-16t^2 + 128t = 0\)
       \[-16t(t - 8) = 0\]
       \(-16t = 0 \Rightarrow t = 0\)
       \(t - 8 = 0 \Rightarrow t = 8\)
   It will be back on the ground in 8 seconds.
   (b) \(-16t^2 + 128t < 128\)
       \(-16t^2 + 128t - 128 < 0\)
       Critical numbers: \(4 - 2\sqrt{2}, 4 + 2\sqrt{2}\)
   Test intervals:
   
   \((-∞, 4 - 2\sqrt{2}), (4 - 2\sqrt{2}, 4 + 2\sqrt{2}), (4 + 2\sqrt{2}, ∞)\)
   Solution set: 0 seconds \(≤ t < 4 - 2\sqrt{2}\) seconds
   and 4 - \(2\sqrt{2}\) seconds \(≤ t ≤ 8\) seconds

69. \( 2L + 2W = 100 \Rightarrow W = 50 - L \)
       \(LW ≥ 500\)
       \(L(50 - L) ≥ 500\)
       \(-L^2 + 50L - 500 ≥ 0\)
   By the Quadratic Formula we have:
   Critical numbers: \(L = 25 ± 5\sqrt{5}\)
   Test: Is \(-L^2 + 50L - 500 ≥ 0\)?
   Solution set: \(25 - 5\sqrt{5} ≤ L ≤ 25 + 5\sqrt{5}\)
   13.8 meters \(≤ L ≤ 36.2\) meters

70. \( 2L + 2W = 440 \Rightarrow W = 220 - L \)
       \(L(220 - L) ≥ 8000\)
       \(-L^2 + 220L - 8000 ≥ 0\)
   By the Quadratic Formula we have:
   Critical numbers: \(L = 110 ± 10\sqrt{41}\)
   Test: Is \(-L^2 + 220L - 8000 ≥ 0\)?
   Solution set: \(110 - 10\sqrt{41} ≤ L ≤ 110 + 10\sqrt{41}\)
   45.97 feet \(≤ L ≤ 174.03\) feet

71. \( R = x(75 - 0.0005x) \) and \( C = 30x + 250,000 \)
   \( P = R - C \)
   \( = (75x - 0.0005x^2) - (30x + 250,000) \)
   \( = -0.0005x^2 + 45x - 250,000 \)
   \( P ≥ 750,000 \)
   \(-0.0005x^2 + 45x - 250,000 ≥ 750,000 \)
   \(-0.0005x^2 + 45x - 1,000,000 ≥ 0 \)
   Critical numbers: \(x = 40,000, x = 50,000 \) (These were obtained by using the Quadratic Formula.)
   Test intervals: \((0, 40,000), (40,000, 50,000), (50,000, ∞)\)
   By testing \(x\)-values in each test interval in the inequality, we see that the solution set is \([40,000, 50,000]\) or \(40,000 ≤ x ≤ 50,000\). The price per unit is
   \( p = \frac{R}{x} = 75 - 0.0005x. \)
   For \(x = 40,000, p = $55.\) For \(x = 50,000, p = $50.\)
   Therefore, for \(40,000 ≤ x ≤ 50,000, $50.00 ≤ p ≤ $55.00.\)

72. What is the price per unit?
   When \(x = 90,000: \)
   \( R = $2,880,000 \Rightarrow \frac{2,880,000}{90,000} = $32\) per unit
   When \(x = 100,000: \)
   \( R = $3,000,000 \Rightarrow \frac{3,000,000}{100,000} = $30\) per unit
   Solution interval: \( $30.00 ≤ p ≤ $32.00 \)
73. \( C = 0.0031t^3 - 0.216t^2 + 5.54t + 19.1, \ 0 \leq t \leq 23 \)

(a) \[ C \] will be greater than 75% when \( t \approx 31 \), which corresponds to 2011.

(b) | \( t \) | \( C \) |
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>24</td>
<td>70.5</td>
</tr>
<tr>
<td>26</td>
<td>71.6</td>
</tr>
<tr>
<td>28</td>
<td>72.9</td>
</tr>
<tr>
<td>30</td>
<td>74.6</td>
</tr>
<tr>
<td>32</td>
<td>76.8</td>
</tr>
<tr>
<td>34</td>
<td>79.6</td>
</tr>
</tbody>
</table>

(c) \( C = 75 \) when \( t = 30.41 \).

(d) \( C \) will be between 85% and 100% when \( t \) is between 37 and 42. These values correspond to the years 2017 to 2022.

(e) \( 85 \leq C \leq 100 \) when \( 36.82 \leq t \leq 41.89 \) or \( 37 \leq t \leq 42 \).

(f) The model is a third-degree polynomial and as \( t \to \infty, \ C \to \infty \).

74. (a) \[ \begin{array}{c|cccccc}
\text{d} & 4 & 6 & 8 & 10 & 12 & \\
\hline
\text{Load} & 2223.9 & 5593.9 & 10,312 & 16,378 & 23,792 & \\
\end{array} \]

| \( d \) | 4 | 6 | 8 | 10 | 12 | \\
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>2223.9</td>
<td>5593.9</td>
<td>10,312</td>
<td>16,378</td>
<td>23,792</td>
</tr>
</tbody>
</table>

(b) \[ 2000 \leq 168.5d^2 - 472.1 \]
\[ 2472.1 \leq 168.5d^2 \]
\[ 14.67 \leq d^2 \]
\[ 3.83 \leq d \]

The minimum depth is 3.83 inches.

75. \[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{2} \]
\[ 2R_1 = 2R + RR_1 \]
\[ 2R_1 = R(2 + R_1) \]
\[ \frac{2R_1}{2 + R_1} = R \]

Since \( R \geq 1 \), we have
\[ \frac{2R_1}{2 + R_1} \geq 1 \]
\[ \frac{2R_1}{2 + R_1} - 1 \geq 0 \]
\[ \frac{R_1 - 2}{2 + R_1} \geq 0. \]

Since \( R_1 > 0 \), the only critical number is \( R_1 = 2 \). The inequality is satisfied when \( R_1 \geq 2 \) ohms.

76. (a) \[ N = -0.03t^2 + 9.6t + 172 \]
\[ = 220 \implies t = 5 \]

So the number of master’s degrees earned by women exceeded 220,000 in 1995.

(c) \[ N = -0.03t^2 + 9.6t + 172 \]
\[ = 320 \implies t = 16.2 \]

So the number of master’s degrees earned by women will exceed 320,000 in 2006.
77. True
\[ x^3 - 2x^2 - 11x + 12 = (x + 3)(x - 1)(x - 4) \]
The test intervals are \((-\infty, -3), (-3, 1), (1, 4), \text{ and } (4, \infty)\).

79. \[ x^2 + bx + 4 = 0 \]
To have at least one real solution, \(b^2 - 16 \geq 0\). This occurs when \(b \leq -4\) or \(b \geq 4\). This can be written as \((-\infty, -4] \cup [4, \infty)\).

81. \[ 3x^2 + bx + 10 = 0 \]
To have at least one real solution, \(b^2 - 4(3)(10) \geq 0\).

83. (a) If \(a > 0\) and \(c \leq 0\), then \(b\) can be any real number. If \(a > 0\) and \(c > 0\), then for \(b^2 - 4ac\) to be greater than or equal to zero, \(b\) is restricted to \(b < -2\sqrt{ac}\) or \(b > 2\sqrt{ac}\).

(b) The center of the interval for \(b\) in Exercises 79–82 is 0.

85. \[ 4x^2 + 20x + 25 = (2x + 5)^2 \]

87. \[ x^2(x + 3) - 4(x + 3) = (x^2 - 4)(x + 3) \]
\[ = (x + 2)(x - 2)(x + 3) \]

89. Area = (length)(width)
\[ = (2x + 1)(x) \]
\[ = 2x^2 + x \]

88. \[ 2x^4 - 54x = 2x(x^3 - 27) \]
\[ = 2x(x - 3)(x^2 + 3x + 9) \]

90. Area = \(\frac{1}{2}(\text{base})(\text{height})\)
\[ = \frac{1}{2}(b)(3b + 2) \]
\[ = \frac{3}{2}b^2 + b \]

78. True
The \(y\)-values are greater than zero for all values of \(x\).

80. \[ x^2 + bx - 4 = 0 \]
To have at least one real solution,
\[ b^2 - 4(1)(-4) \geq 0 \]
\[ b^2 + 16 \geq 0. \]
This inequality is true for all real values of \(b\). Thus, the interval for \(b\) such that the equation has at least one real solution is \((-\infty, \infty)\).

82. \[ 2x^2 + bx + 5 = 0 \]
To have at least one real solution,
\[ b^2 - 4(2)(5) \geq 0 \]
\[ b^2 - 40 \geq 0. \]
This occurs when \(b \leq -2\sqrt{10}\) or \(b \geq 2\sqrt{10}\). Thus, the interval for \(b\) such that the equation has at least one real solution is \((-\infty, -2\sqrt{10}] \cup [2\sqrt{10}, \infty)\).

84. (a) \(x = a, x = b\)
(b) \(-\quad \ast \quad \ast \quad +\)
\(-\ast\quad\ast\quad +\)
\(+\ast\quad\ast\quad +\)
\[ a\quad\quad b\quad\quad x \]

86. \[ (x + 3)^2 - 16 = [(x + 3) + 4][(x + 3) - 4] \]
\[ = (x + 7)(x - 1) \]
Review Exercises for Chapter 1

1. \( y = 3x - 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-11</td>
<td>-8</td>
<td>-5</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

2. \( y = -\frac{1}{2}x + 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

3. \( y = x^2 - 3x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

4. \( y = 2x^2 - x - 9 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>-6</td>
<td>-9</td>
<td>-8</td>
<td>-3</td>
<td>6</td>
</tr>
</tbody>
</table>

5. \( y - 2x - 3 = 0 \)

\( y = 2x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Line with \( x \)-intercept \((-\frac{3}{2}, 0)\) and \( y \)-intercept \((0, 3)\)

6. \( 3x + 2y + 6 = 0 \)

\( 2y = -3x - 6 \)

\( y = -\frac{3}{2}x - 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(-\frac{3}{2})</td>
<td>0</td>
<td>-(\frac{3}{2})</td>
<td>-3</td>
<td>-(\frac{9}{2})</td>
</tr>
</tbody>
</table>

Line with \( x \)-intercept \((-2, 0)\) and \( y \)-intercept \((0, -3)\)

7. \( y = \sqrt{5 - x} \)

Domain: \((-\infty, 5]\)

<table>
<thead>
<tr>
<th>( x )</th>
<th>5</th>
<th>4</th>
<th>1</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
8. \( y = \sqrt{x + 2} \), domain: \([-2, \infty)\)

9. \( y + 2x^2 = 0 \)

10. \( y = x^2 - 4x \) is a parabola.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>( \sqrt{2} )</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c|cccc}
 x & -1 & 0 & 1 & 2 \\
 y & 5 & 0 & -3 & -4 \\
\end{array}
\]

11. \( y = (x - 3)^2 - 4 \)

\( x \)-intercepts: \( 0 = (x - 3)^2 - 4 \Rightarrow (x - 3)^2 = 4 \Rightarrow x - 3 = \pm 2 \Rightarrow x = 3 \pm 2 \)

\( y \)-intercept: \( y = (0 - 3)^2 - 4 \Rightarrow y = 9 - 4 \Rightarrow y = 5 \)

The \( x \)-intercepts are \((1, 0)\) and \((5, 0)\). The \( y \)-intercept is \((0, 5)\).

12. \( y = |x + 1| - 3 \)

\( 0 = |x + 1| - 3 \)

For \( x + 1 > 0 \), \( 0 = x + 1 - 3 \), or \( 2 = x \).

For \( x + 1 < 0 \), \( 0 = -(x + 1) - 3 \), or \( -4 = x \).

\( y = |x + 1| - 3 \)

\( y = |0 + 1| - 3 \) or \( y = -2 \)

The \( x \)-intercepts are \((2, 0)\) and \((-4, 0)\); the \( y \)-intercept is \((0, -2)\).

13. \( y = -4x + 1 \)

Intercepts: \((\frac{1}{2}, 0)\), \((0, 1)\)

\( y = -4(-x) + 1 \Rightarrow y = 4x + 1 \Rightarrow \) No \( y \)-axis symmetry

\( y = -4x + 1 \Rightarrow y = 4x - 1 \Rightarrow \) No \( x \)-axis symmetry

\( y = -4(-x) + 1 \Rightarrow y = -4x - 1 \Rightarrow \) No origin symmetry

14. \( y = 5x - 6 \)

Intercepts: \((\frac{6}{5}, 0)\), \((0, -6)\)

No symmetry
15. \( y = 5 - x^2 \)

Intercepts: \((\pm \sqrt{3}, 0), (0, 5)\)

\( y = 5 - (-x)^2 \Rightarrow y = 5 - x^2 \Rightarrow y\)-axis symmetry

\(-y = 5 - x^2 \Rightarrow y = -5 + x^2 \Rightarrow \) No \(x\)-axis symmetry

\(-y = 5 - (-x)^2 \Rightarrow y = -5 + x^2 \Rightarrow \) No origin symmetry

16. \( y = x^2 - 10 \)

Intercepts: \((\pm \sqrt{10}, 0), (0, -10)\)

\( y\)-axis symmetry

17. \( y = x^3 + 3 \)

Intercepts: \((-\sqrt{3}, 0), (0, 3)\)

\( y = (-x)^3 + 3 \Rightarrow y = -x^3 + 3 \Rightarrow \) No \(y\)-axis symmetry

\(-y = x^3 + 3 \Rightarrow y = -x^3 - 3 \Rightarrow \) No \(x\)-axis symmetry

\(-y = (-x)^3 + 3 \Rightarrow y = x^3 - 3 \Rightarrow \) No origin symmetry

18. \( y = -6 - x^3 \)

Intercepts: \((\sqrt[3]{-6}, 0), (0, -6)\)

No symmetry

19. \( y = \sqrt{x + 5} \)

Domain: \([-5, \infty)\)

Intercepts: \((-5, 0), (0, \sqrt{5})\)

\( y = \sqrt{-x + 5} \Rightarrow \) No \(y\)-axis symmetry

\(-y = \sqrt{x + 5} \Rightarrow y = -\sqrt{x + 5} \Rightarrow \) No \(x\)-axis symmetry

\(-y = \sqrt{-x + 5} \Rightarrow y = -\sqrt{-x + 5} \Rightarrow \) No origin symmetry

20. \( y = |x| + 9 \)

Intercepts: \((0, 9)\)

\( y\)-axis symmetry

21. \( x^2 + y^2 = 9 \)

Center: \((0, 0)\)

Radius: 3
22. \( x^2 + y^2 = 4 \)
   Center: \((0, 0)\)
   Radius: 2

23. \( (x + 2)^2 + y^2 = 16 \)
   \( (x - (-2))^2 + (y - 0)^2 = 4^2 \)
   Center: \((-2, 0)\)
   Radius: 4

24. \( x^2 + (y - 8)^2 = 81 \)
   Center: \((0, 8)\)
   Radius: 9

25. \( \left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = 36 \)
   \( \left(x - \frac{1}{2}\right)^2 + (y - (-1))^2 = 6^2 \)
   Center: \(\left(\frac{1}{2}, -1\right)\)
   Radius: 6

26. \( (x + 4)^2 + \left(y - \frac{3}{2}\right)^2 = 100 \)
   \( (x - (-4))^2 + \left(y - \frac{3}{2}\right)^2 = 100 \)
   Center: \((-4, \frac{3}{2})\)
   Radius: 10

27. Endpoints of a diameter: \((0, 0)\) and \((4, -6)\)
   Center: \(\left(\frac{0 + 4}{2}, \frac{0 + (-6)}{2}\right) = (2, -3)\)
   Radius: \(r = \sqrt{(2 - 0)^2 + (-3 - 0)^2} = \sqrt{4 + 9} = \sqrt{13}\)
   Standard form: \((x - 2)^2 + (y + 3)^2 = 13\)

28. Endpoints of a diameter: \((-2, -3)\) and \((4, -10)\)
   Center: \(\left(\frac{-2 + 4}{2}, \frac{-3 + (-10)}{2}\right) = (1, -\frac{13}{2})\)
   Radius: \(r = \sqrt{(1 - (-2))^2 + \left(-\frac{13}{2} - (-3)\right)^2} = \sqrt{9 + \frac{49}{4}} = \sqrt{\frac{85}{4}}\)
   Standard form: \((x - 1)^2 + \left(y - \left(-\frac{13}{2}\right)\right)^2 = \left(\sqrt{\frac{85}{4}}\right)^2\)

29. \( F = \frac{5}{2}x, \ 0 \leq x \leq 20 \)
   (a) \[
   \begin{array}{cccccc}
   x & 0 & 4 & 8 & 12 & 16 & 20 \\
   F & 0 & 5 & 10 & 15 & 20 & 25 \\
   \end{array}
   \]
   (c) When \( x = 10, \ F = \frac{50}{2} = 25 \) pounds.
30. (a) 

![Graph showing the number of Target stores over years](image)

(b) \( z = 9.94; \) The number of stores was \( 1300 \) in 2003.

32. \( 3(x - 2) + 2x = 2(x + 3) \)

\[
3x - 6 + 2x = 2x + 6
\]

\[
x = 12
\]

Conditional equation

34. \( 3(x^2 - 4x + 8) = -10(x + 2) - 3x^2 + 6 \)

\[
3x^2 - 12x + 24 = -10x - 20 - 3x^2 + 6
\]

\[
3x^2 - 12x + 24 = -3x^2 - 10x - 14
\]

\[
6x^2 - 2x + 38 = 0
\]

\[
3x^2 - x + 19 = 0
\]

Conditional equation

36. \( 4x + 2(7 - x) = 5 \)

\[
4x + 14 - 2x = 5
\]

\[
2x = -9
\]

\[
x = \frac{-9}{2}
\]

39. \( \frac{x}{5} - 3 = \frac{2x}{2} + 1 \)

\[
5\left(\frac{x}{5} - 3\right) = (x + 1)5
\]

\[
x - 15 = 5x + 5
\]

\[
-4x = 20
\]

\[
x = -5
\]

42. \( \frac{5}{x - 2} = \frac{13}{2x - 3} \)

\[
5(2x - 3) = 13(x - 2)
\]

\[
10x - 15 = 13x - 26
\]

\[
-3x = -11
\]

\[
x = \frac{11}{3}
\]

31. \( 6 - (x - 2)^2 = 2 + 4x - x^2 \)

\[
6 - (x^2 - 4x + 4) = 2 + 4x - x^2
\]

\[
2 + 4x - x^2 = 2 + 4x - x^2
\]

\[
0 = 0 \quad \text{Identity}
\]

All real numbers are solutions.

33. \( -x^3 + x(7 - x) + 3 = x(-x^2 - x) + 7(x + 1) - 4 \)

\[
-x^3 + 7x - x^2 + 3 = -x^3 - x^2 + 7x + 7 - 4
\]

\[
-x^3 - x^2 + 7x + 3 = -x^3 - x^2 + 7x + 3
\]

\[
0 = 0 \quad \text{Identity}
\]

All real numbers are solutions.

35. \( 3x - 2(x + 5) = 10 \)

\[
3x - 2x - 10 = 10
\]

\[
x = 20
\]

37. \( 4(x + 3) - 3 = 2(4 - 3x) - 4 \)

\[
4x + 12 - 3 = 8 - 6x - 4
\]

\[
4x + 9 = -6x + 4
\]

\[
10x = -5
\]

\[
x = -\frac{1}{2}
\]

40. \( \frac{4x - 3}{6} + \frac{x}{4} = x - 2 \)

\[
2(4x - 3) + 3x = 12x - 24
\]

\[
8x - 6 + 3x = 12x - 24
\]

\[
-x = -18
\]

\[
x = 18
\]

41. \( \frac{18}{x} = \frac{10}{x - 4} \)

\[
18(x - 4) = 10x
\]

\[
18x - 72 = 10x
\]

\[
x = 72
\]

43. \( y = 3x - 1 \)

\( x\)-intercept: \( 0 = 3x - 1 \implies x = \frac{1}{3} \)

\( y\)-intercept: \( y = 3(0) - 1 \implies y = -1 \)

The \( x\)-intercept is \( \left(\frac{1}{3}, 0\right) \) and the \( y\)-intercept is \( (0, -1) \).
44. \( y = -5x + 6 \)
\[
\begin{align*}
0 &= -5x + 6 \\
-6 &= -5x \\
\frac{6}{5} &= x
\end{align*}
\]
The x-intercept is \((\frac{6}{5}, 0)\) and the y-intercept is (0, 6).

45. \( y = 2(x - 4) \)
\[
\begin{align*}
x\text{-intercept: } 0 &= 2(x - 4) \implies x = 4 \\
y\text{-intercept: } y &= 2(0 - 4) \implies y = -8
\end{align*}
\]
The x-intercept is (4, 0) and the y-intercept is (0, -8).

46. \( y = 4(7x + 1) \)
\[
\begin{align*}
0 &= 4(7x + 1) \\
0 &= 28x + 4 \\
-4 &= 28x \\
\frac{-1}{7} &= x
\end{align*}
\]
The x-intercept is \((-\frac{1}{7}, 0)\) and the y-intercept is (0, 4).

47. \( y = -\frac{1}{2}x + \frac{2}{3} \)
\[
\begin{align*}
x\text{-intercept: } 0 &= -\frac{1}{2}x + \frac{2}{3} \implies x = \frac{2/3}{1/2} = \frac{4}{3} \\
y\text{-intercept: } y &= -\frac{1}{2}(0) + \frac{2}{3} \implies y = \frac{2}{3}
\end{align*}
\]
The x-intercept is \((\frac{4}{3}, 0)\) and the y-intercept is \((0, \frac{2}{3})\).

48. \( y = \frac{3}{4}x - \frac{1}{4} \)
\[
\begin{align*}
0 &= \frac{3}{4}x - \frac{1}{4} \\
\frac{4}{3} \cdot \frac{1}{4} &= \frac{3}{4}x - \frac{1}{4} \\
\frac{1}{3} &= x
\end{align*}
\]
The x-intercept is \((\frac{1}{3}, 0)\) and the y-intercept is \((0, -\frac{1}{4})\).

49. \( 3.8y - 0.5x + 1 = 0 \)
\[
\begin{align*}
x\text{-intercept: } 3.8(0) - 0.5x + 1 &= 0 \implies x = \frac{1}{0.5} = 2 \\
y\text{-intercept: } 3.8y - 0.5(0) + 1 &= 0 \implies y = -\frac{1}{3.8} = -\frac{5}{19}
\end{align*}
\]
The x-intercept is (2, 0) and the y-intercept is \((0, -\frac{5}{19})\).

50. \( 1.5y + 2x - 1.2 = 0 \)
\[
\begin{align*}
1.5(0) + 2x - 1.2 &= 0 \\
2x &= 1.2 \\
x &= 0.6
\end{align*}
\]
The x-intercept is \((0.6, 0)\) and the y-intercept is \((0, 0.8)\).

51. \( 244.92 = 2(3.14)(3)^2 + 2(3.14)(3)h \)
\[
\begin{align*}
244.92 &= 56.52 + 18.84h \\
188.40 &= 18.84h \\
10 &= h
\end{align*}
\]
The height is 10 inches.

52. \( C = \frac{5}{9}F - \frac{160}{9} \)
\[
\begin{align*}
\frac{5}{9}F &= C + \frac{160}{9} \\
F &= \frac{9}{5}(C + \frac{160}{9})
\end{align*}
\]
For \( C = 100^\circ \): \( F = \frac{9}{5}(100 + \frac{160}{9}) = 212^\circ F \)

53. **Verbal Model:** September’s profit + October’s profit = 689,000

**Labels:** Let \( x = \) September’s profit. Then \( x + 0.12x = \) October’s profit.

**Equation:** \( x + (x + 0.12x) = 689,000 \)
\[
\begin{align*}
2.12x &= 689,000 \\
x &= 325,000 \\
x + 0.12x &= 364,000
\end{align*}
\]
**Answer:** September profit: $325,000, October profit: $364,000
54. **Model:** \((\text{Original price})(1 - \text{discount rate}) = \text{(sale price)}\)

**Labels:**
- Original price = 340 + 85 = 425
- Discount rate = \(x\)
- Sale price = 340

**Equation:**
\[425(1 - x) = 340\]
\[1 - x = 0.8\]
\[x = 0.2\]

The percent discount is 20%.

55. Let \(x = \text{height of the streetlight.}\)

By similar triangles we have:
\[
\frac{x}{20} = \frac{6}{5}
\]
\[5x = 120\]
\[x = 24\]

The streetlight is 24 feet tall.

56. Let \(x = \text{the number of members in the group.}\)

Cost per member = \(\frac{48,000}{x}\)

If two more members join the group, the cost per member will be \(\frac{48,000}{x + 2}\)

Since this new cost is $4000 less than the original cost:
\[
\frac{48,000}{x} - 4000 = \frac{48,000}{x + 2}
\]
\[48,000(x + 2) - 4000(x + 2) = 48,000x\]
\[12(x + 2) - x(x + 2) = 12x\quad \text{Divide both sides by 4000.}\]
\[12x + 24 - x^2 - 2x = 12x\]
\[0 = x^2 + 2x - 24\]
\[0 = (x + 6)(x - 4)\]
\[x + 6 = 0 \Rightarrow x = -6, \text{ extraneous}\]
\[x - 4 = 0 \Rightarrow x = 4\]

There are four members presently in the group.

57. Let \(x = \text{the number of original investors.}\)

Each person’s share is \(\frac{90,000}{x}\). If three more people invest, each person’s share is \(\frac{90,000}{x + 3}\).

Since this is $2500 less than the original cost, we have:
\[
\frac{90,000}{x} - 2500 = \frac{90,000}{x + 3}
\]
\[90,000(x + 3) - 2500x(x + 3) = 90,000x\]
\[90,000x + 270,000 - 2500x^2 - 7500x = 90,000x\]
\[-2500x^2 - 7500x + 270,000 = 0\]
\[-2500(x^2 + 3x - 108) = 0\]
\[-2500(x + 12)(x - 9) = 0\]
\[x = -12, \text{ extraneous} \quad \text{or} \quad x = 9\]

There are currently nine investors.
58. 

<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>$\frac{56}{r}$</td>
<td>56</td>
</tr>
<tr>
<td>$r + 8$</td>
<td>$\frac{56}{r + 8}$</td>
<td>56</td>
</tr>
</tbody>
</table>

Time = \frac{\text{distance}}{\text{rate}}

Time to work = time from work + 10 minutes

\[
6(\frac{56}{r + 8}) = 6r + 8 \quad \text{Convert minutes to portion of an hour.}
\]

\[
336r + 2688 = 336r + r^2 + 8r
\]

\[
0 = r^2 + 8r - 2688
\]

\[
0 = (r - 48)(r + 56)
\]

Using the positive value for $r$, we have $r = 48$ miles per hour. The average speed on the trip home was $r + 8 = 56$ miles per hour.

59. Let $x$ = the number of liters of pure antifreeze.

30% of $(10 - x) + 100\%$ of $x = 50\%$ of 10

\[
0.30(10 - x) + 1.00x = 0.50(10)
\]

\[
3 - 0.30x + 1.00x = 5
\]

\[
0.70x = 2
\]

\[
x = \frac{2}{0.70} = \frac{20}{7} = 2.86 \text{ liters}
\]

60. Model: \((\text{Interest from } \frac{4\%}{2}) + (\text{interest from } \frac{5\%}{2}) = (\text{total interest})\n
Labels: Amount invested at $\frac{4\%}{2} = x$, amount invested at $\frac{5\%}{2} = 6000 - x$

Interest from $\frac{4\%}{2} = x(0.045)(1)$, interest from $\frac{5\%}{2} = (6000 - x)(0.055)(1)$, total interest = $305$

Equation: $0.045x + 0.055(6000 - x) = 305$

\[
0.045x + 330 - 0.055x = 305
\]

\[
-0.01x = -25
\]

\[
x = 2500
\]

The amount invested at $\frac{4\%}{2}$ was $2500$ and the amount invested at $\frac{5\%}{2}$ was $6000 - 2500 = 3500$.

61. \[V = \frac{1}{3}\pi r^2 h\]

\[3V = \pi r^2 h\]

\[\frac{3V}{\pi r^2} = h\]

62. \[E = \frac{1}{2}mv^2\]

\[mv^2 = 2E\]

\[m = \frac{2E}{v^2}\]
63.  
<table>
<thead>
<tr>
<th></th>
<th>rate</th>
<th>time</th>
<th>distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st car</td>
<td>40</td>
<td>t</td>
<td>40t</td>
</tr>
<tr>
<td>2nd car</td>
<td>55</td>
<td>t</td>
<td>55t</td>
</tr>
</tbody>
</table>

\[ 55t - 40t = 10 \]
\[ 15t = 10 \]
\[ t = \frac{1}{2} \text{ hour or 40 minutes} \]

65. \[ 15 + x - 2x^2 = 0 \]
\[ 0 = 2x^2 - x - 15 \]
\[ 0 = (2x + 5)(x - 3) \]
\[ 2x + 5 = 0 \implies x = -\frac{5}{2} \]
\[ x - 3 = 0 \implies x = 3 \]

66. \[ 2x^2 - x - 28 = 0 \]
\[ (2x + 7)(x - 4) = 0 \]
\[ 2x + 7 = 0 \implies x = -\frac{7}{2} \]
\[ x - 4 = 0 \implies x = 4 \]

67. \[ 6 = 3x^2 \]
\[ 2 = x^2 \]
\[ \pm \sqrt{2} = x \]

68. \[ 16x^2 = 25 \]
\[ 16x^2 - 25 = 0 \]
\[ (4x - 5)(4x + 5) = 0 \]
\[ 4x - 5 = 0 \implies x = \frac{5}{4} \]
\[ 4x + 5 = 0 \implies x = -\frac{5}{4} \]

69. \[ (x + 4)^2 = 18 \]
\[ x + 4 = \pm \sqrt{18} \]
\[ x = -4 \pm 3 \sqrt{2} \]

70. \[ (x - 8)^2 = 15 \]
\[ x - 8 = \pm \sqrt{15} \]
\[ x = 8 \pm \sqrt{15} \]

71. \[ x^2 - 12x + 30 = 0 \]
\[ x^2 - 12x = -30 \]
\[ x^2 - 12x + 36 = -30 + 36 \]
\[ (x - 6)^2 = 6 \]
\[ x - 6 = \pm \sqrt{6} \]
\[ x = 6 \pm \sqrt{6} \]

72. \[ x^2 + 6x - 3 = 0 \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-6 \pm \sqrt{6^2 - 4(1)(-3)}}{2(1)} \]
\[ = \frac{-6 \pm \sqrt{48}}{2} \]
\[ = \frac{-6 \pm 4 \sqrt{3}}{2} \]
\[ = -3 \pm 2 \sqrt{3} \]

74. \[ -20 - 3x + 3x^2 = 0 \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)} \]
\[ = \frac{3 \pm \sqrt{249}}{6} = \frac{1}{2} \pm \frac{\sqrt{249}}{6} \]

75. \[ M = 500x(20 - x) \]
(a) \[ 500x(20 - x) = 0 \] when \( x = 0 \) feet and \( x = 20 \) feet.
(b) \[ 55,000 \]
(c) The bending moment is greatest when \( x = 10 \) feet.
76. (a) \( h(t) = -16t^2 + 30t + 5.8 \)
   (b) \( h(1) = -16 \cdot 1^2 + 30 \cdot 1 + 5.8 = 19.8 \) feet
   (c) \(-16t^2 + 30t + 5.8 = 0\
   \[ t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(5.8)}}{2(-16)} \]
   \[ = \frac{-30 \pm \sqrt{1271.2}}{-32} \]
   \[ \approx 2.052 \text{ or } -0.1767 \]

   The ball will hit the ground in about two seconds.

77. \( 6 + \sqrt{-4} = 6 + 2i \)

78. \( 3 - \sqrt{25} = 3 - 5i \)

79. \( i^2 + 3i = -1 + 3i \)

80. \(-5i + i^2 = -1 - 5i \)

81. \((7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i) = 3 + 7i \)

82. \( \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}i \) - \( \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}i \) = \( \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}i - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}i = -2 \left( \frac{\sqrt{3}}{2}i \right) = -\sqrt{2}i \)

83. \( 5i(13 - 8i) = 65i - 40i^2 = 40 + 65i \)

84. \((1 + 6i)(5 - 2i) = 5 - 2i + 30i - 12i^2 = 5 + 28i + 12 = 5 + 28i \)

85. \((10 - 8i)(2 - 3i) = 20 - 30i - 16i + 24i^2 = -4 - 46i \)

86. \(i(6 + i)(3 - 2i) = i(18 - 12i + 3i - 2i^2) = i(20 - 9i) = 20i - 9i^2 = 9 + 20i \)

87. \( \frac{6 + i}{4 - i} = \frac{6 + i}{4 - i} \cdot \frac{4 + i}{4 + i} = \frac{24 + 10i + i^2}{16 + 1} = \frac{24 + 10i}{16 + 1} = \frac{23 + 10i}{17} \)

88. \( \frac{3 + 2i}{5 + i} = \frac{3 + 2i}{5 + i} \cdot \frac{5 - i}{5 - i} = \frac{15 + 10i - 2i^2}{25 - i^2} = \frac{17 + 7i}{26} \)

89. \( \frac{4}{2 - 3i} + \frac{2}{1 + i} = \frac{4}{2 - 3i} + \frac{2}{1 + i} = \frac{8 + 12i}{4 + 9} + \frac{2 - 2i}{1 + 1} = \frac{8 + 12i}{13} + \frac{2 - 2i}{13} = \left( \frac{8}{13} + 1 \right) + \left( \frac{12}{13} - i \right) = \frac{21}{13} - \frac{1}{13}i \)

90. \( \frac{1}{2 + i} - \frac{5}{1 + 4i} = \frac{(1 + 4i) - 5(2 + i)}{(2 + i)(1 + 4i)} = \frac{1 + 4i - 10 - 5i}{2 + 8i + i + 4i^2} = \frac{-9 - i}{2 + 9i} \cdot \frac{(-2 - 9i)}{(-2 - 9i)} = \frac{18 + 81i + 2i + 9i^2}{4 - 81i^2} = \frac{9 + 83i}{85} = \frac{9}{85} + \frac{83i}{85} \)
91. \(3x^2 + 1 = 0\)

\[ 3x^2 = -1 \]
\[ x^2 = -\frac{1}{3} \]
\[ x = \pm \sqrt{-\frac{1}{3}} \]
\[ = \pm \frac{1}{3} \sqrt{-3} i \]

92. \(2 + 8x^2 = 0\)

\[ 8x^2 = -2 \]
\[ x^2 = -\frac{1}{4} \]
\[ x = \pm \frac{i}{2} \]

93. \(x^2 - 2x + 10 = 0\)

\[ (x - 1)^2 = -9 \]
\[ x - 1 = \pm \sqrt{-9} \]
\[ x = 1 \pm 3i \]

94. \(6x^2 + 3x + 27 = 0\)

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-3 \pm \sqrt{3^2 - 4(6)(27)}}{2(6)} \]
\[ = \frac{-3 \pm 3\sqrt{-639}}{12} \]
\[ = -\frac{3}{2} \pm \frac{\sqrt{11}}{2} i \]

95. \(5x^4 - 12x^3 = 0\)

\[ x^3(5x - 12) = 0 \]
\[ x^3 = 0 \text{ or } 5x - 12 = 0 \]
\[ x = 0 \text{ or } x = \frac{12}{5} \]
\[ 4x - 6 = 0 \Rightarrow x = \frac{3}{2} \]

96. \(4x^3 - 6x^2 = 0\)

\[ x^2(4x - 6) = 0 \]
\[ x^2 = 0 \Rightarrow x = 0 \]
\[ 4x - 6 = 0 \Rightarrow x = \frac{3}{2} \]

97. \(x^4 - 5x^3 + 6 = 0\)

\[(x^2 - 2)(x^2 - 3) = 0 \]
\[ x^2 - 2 = 0 \text{ or } x^2 - 3 = 0 \]
\[ x^2 = 2 \text{ or } x^2 = 3 \]
\[ x = \pm \sqrt{2} \text{ or } x = \pm \sqrt{3} \]

98. \(9x^4 + 27x^3 - 4x^2 - 12x = 0\)

\[ 9x^2(x + 3) - 4x(x + 3) = 0 \]
\[ (x + 3)(9x^3 - 4x) = 0 \]
\[ (x + 3)(9x^2 - 4) = 0 \]
\[ x + 3 = 0 \Rightarrow x = -3 \]
\[ x = 0 \]
\[ 9x^2 - 4 = 0 \Rightarrow x = \pm \frac{2}{3} \]

99. \(\sqrt{x + 4} = 3\)

\[(\sqrt{x + 4})^2 = (3)^2 \]
\[ x + 4 = 9 \]
\[ x = 5 \]

100. \(\sqrt{x - 2} - 8 = 0\)

\[ \sqrt{x - 2} = 8 \]
\[ x - 2 = 64 \]
\[ x = 66 \]

101. \(\sqrt{2x + 3} + \sqrt{x - 2} = 2\)

\[(\sqrt{2x + 3})^2 = (2 - \sqrt{x - 2})^2 \]
\[ 2x + 3 = 4 - 4\sqrt{x - 2} + x - 2 \]
\[ x + 1 = -4\sqrt{x - 2} \]
\[ (x + 1)^2 = (-4\sqrt{x - 2})^2 \]
\[ x^2 + 2x + 1 = 16(x - 2) \]
\[ x^2 - 14x + 33 = 0 \]
\[ (x - 3)(x - 11) = 0 \]
\[ x = 3, \text{ extraneous or } x = 11, \text{ extraneous} \]

No solution
102. \( 5 \sqrt{x} - \sqrt{x - 1} = 6 \)

\[
5 \sqrt{x} = 6 + \sqrt{x - 1} \\
25x = 36 + 12 \sqrt{x - 1} + x - 1 \\
24x - 35 = 12 \sqrt{x - 1} \\
576x^2 - 1680x + 1225 = 144(x - 1) \\
576x^2 - 1824x + 1369 = 0
\]

\[
x = \frac{-(-1824) \pm \sqrt{(-1824)^2 - 4(576)(1369)}}{2(576)} = \frac{1824 \pm \sqrt{172800}}{1152} = \frac{1824 \pm 240\sqrt{3}}{1152}
\]

\[
x = \frac{38 + 5\sqrt{3}}{24} \\
x = \frac{38 - 5\sqrt{3}}{24}, \text{ extraneous}
\]

103. \((x - 1)^{2/3} - 25 = 0\)

\[
(x - 1)^{2/3} = 25 \\
(x - 1)^2 = 25^3 \\
x - 1 = \pm \sqrt{25^3} \\
x = 1 \pm 125 \\
x = 126 \text{ or } x = -124
\]

104. \((x + 2)^{3/4} = 27\)

\[
x + 2 = 27^{4/3} \\
x + 2 = 81 \\
x = 79
\]

105. \((x + 4)^{1/2} + 5(x + 4)^{3/2} = 0\)

\[
(x + 4)^{1/2}[1 + 5(x + 4)] = 0 \\
(x + 4)^{1/2}(5x^2 + 20x + 1) = 0 \\
(x + 4)^{1/2} = 0 \text{ or } 5x^2 + 20x + 1 = 0 \\
x = -4
\]

\[
x = -\frac{20 \pm \sqrt{400 - 20}}{10} \\
x = -\frac{20 \pm \sqrt{400 - 20}}{10} \\
x = -\frac{20 \pm 2\sqrt{95}}{10} \\
x = -2 \pm \frac{\sqrt{95}}{5}
\]

106. \(8x^2(x^2 - 4)^{1/3} + (x^2 - 4)^{4/3} = 0\)

\[
(x^2 - 4)^{1/3}[8x^2 + x^2 - 4] = 0 \\
(x^2 - 4)^{1/3}(9x^2 - 4) = 0 \\
(x - 2)^{1/3}(x + 2)^{1/3}(3x - 2)(3x + 2) = 0 \\
x - 2 = 0 \implies x = 2 \\
x + 2 = 0 \implies x = -2 \\
3x - 2 = 0 \implies x = \frac{2}{3} \\
3x + 2 = 0 \implies x = -\frac{2}{3}
\]

107. \(\frac{5}{x} = 1 + \frac{3}{x + 2}\)

\[
5(x + 2) = (x)(x + 2) + 3x \\
5x + 10 = x^2 + 2x + 3x \\
10 = x^2 \\
\pm \sqrt{10} = x
\]
108. \( \frac{6(x+5)}{x} + x(x+5) - \frac{8}{x+5} = 3x(x+5) \)

\( 6(x+5) + 8x = 3x(x+5) \)
\( 14x + 30 = 3x^2 + 15x \)
\( 0 = 3x^2 + x - 30 \)
\( 0 = (3x + 10)(x - 3) \)
\( 0 = 3x + 10 \Rightarrow x = -\frac{10}{3} \)
\( 0 = x - 3 \Rightarrow x = 3 \)

110. \( \frac{x(x+5)}{x+5} + \frac{x(x+5)}{x} = \frac{x(x+5)20}{x} \)

\( 12x + 5x + 25 = 20x + 100 \)
\( 17x + 25 = 20x + 100 \)
\( -3x = 75 \)
\( x = -25 \)

112. \( |2x + 3| = 7 \)

\( 2x + 3 = 7 \) or \( 2x + 3 = -7 \)
\( 2x = 4 \quad 2x = -10 \)
\( x = 2 \quad x = -5 \)

114. \( |x^2 - 6| = x \)

\( x^2 - 6 = x \) or \( -(x^2 - 6) = x \)
\( x^2 - x - 6 = 0 \)
\( (x - 3)(x + 2) = 0 \)
\( x - 3 = 0 \Rightarrow x = 3 \)
\( x + 2 = 0 \Rightarrow x = -2 \), extraneous

115. \( 29.95 = 42 - \sqrt{0.001x + 2} \)

\( -12.05 = -\sqrt{0.001x + 2} \)
\( \sqrt{0.001x + 2} = 12.05 \)
\( 0.001x + 2 = 145.2025 \)
\( 0.001x = 143.2025 \)
\( x = 143.202.5 \)
\( \approx 143,203 \) units

109. \( \frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x} \)

\( 3(5x) - 5(x + 2) = x + 2 \)
\( 15x - 5x - 10 = x + 2 \)
\( 9x = 12 \)
\( x = \frac{4}{3} \)

111. \( |x - 5| = 10 \)

\( x - 5 = -10 \) or \( x - 5 = 10 \)
\( x = -5 \quad x = 15 \)

113. \( |x^2 - 3| = 2x \)

\( x^2 - 3 = 2x \) or \( x^2 - 3 = -2x \)
\( x^2 - 2x - 3 = 0 \)
\( x^2 + 2x - 3 = 0 \)
\( (x - 3)(x + 1) = 0 \)
\( (x + 3)(x - 1) = 0 \)
\( x = 3 \) or \( x = -1 \)
\( x = -3 \) or \( x = 1 \)

The only solutions to the original equation are \( x = 3 \) or \( x = 1 \). (\( x = -3 \) and \( x = -1 \) are extraneous.)

116. (a) [Graph image]

(b) There were 800 daily evening papers about 1997.

(c) \( 800 = 1481 - 4.6t^{3/2} \)
\( -681 = -4.6t^{3/2} \)
\( t^{3/2} = \frac{-681}{-4.6} = 148.04 \)
\( t \approx (148.04)^{2/3} \)
\( t \approx 27.98 \Rightarrow 1998 \)
117. Interval: \((-7, 2]\)
Inequality: \(-7 < x \leq 2\)
The interval is bounded.

118. Interval: \((4, \infty)\)
Inequality: \(4 < x < \infty\)
The interval is bounded.

119. Interval: \((-\infty, -10]\)
Inequality: \(x \leq -10\)
The interval is bounded.

120. Interval: \([-2, 2]\)
Inequality: \(-2 \leq x \leq 2\)
The interval is bounded.

121. \(9x - 8 \leq 7x + 16\)
\(2x \leq 24\)
\(x \leq 12\)
\((-\infty, 12]\)
\((-2, \infty)

122. \(\frac{15}{2}x + 4 > 3x - 5\)
\(\frac{9}{2}x > -9\)
\(9x > -18\)
\(x > -2\)

123. \(4(5 - 2x) \leq \frac{1}{3}(8 - x)\)
\(20 - 8x \leq 4 - \frac{1}{3}x\)
\(-\frac{14}{3}x \leq -16\)
\(x \geq \frac{32}{13}\)
\([\frac{32}{13}, \infty)\)

124. \(\frac{1}{2}(3 - x) > \frac{1}{2}(2 - 3x)\)
\(9 - 3x > 4 - 6x\)
\(3x > -5\)
\(x > -\frac{5}{3}\)
\((-\frac{5}{3}, \infty)\)

125. \(-19 < 3x - 17 \leq 34\)
\(-2 < 3x \leq 51\)
\(-\frac{2}{3} < x \leq 17\)
\((-\frac{2}{3}, 17]\)

126. \(-3 \leq \frac{2x - 5}{3} < 5\)
\(-9 \leq 2x - 5 < 15\)
\(-4 \leq 2x < 20\)
\(-2 \leq x < 10\)
\([-2, 10)\)

127. \(|x| \leq 4\)
\(-4 \leq x \leq 4\)

128. \(|x - 2| < 1\)
\(-1 < x - 2 < 1\)
\(1 < x < 3\)

129. \(|x - 3| > 4\)
\(x - 3 < -4\) or \(x - 3 > 4\)
\(x < -1\) or \(x > 7\)
\((-\infty, -1) \cup (7, \infty)\)

130. \(|x - \frac{3}{2}| \geq \frac{1}{2}\)
\(x - \frac{3}{2} \leq -\frac{1}{2}\) or \(x - \frac{3}{2} \geq \frac{1}{2}\)
\(x \leq 0\) or \(x \geq 3\)
\((-\infty, 0] \cup [3, \infty)\)

131. If the side is 19.3 cm, then with the possible error of 0.5 cm we have:
\(18.8 \leq \text{side} \leq 19.8\)
\(353.44 \text{ cm}^2 \leq \text{area} \leq 392.04 \text{ cm}^2\)

132. \(125.33x > 92x + 1200\)
\(33.33x > 1200\)
\(x > 36\) units

133. \(x^2 - 6x - 27 < 0\)
\((x + 3)(x - 9) < 0\)
Critical numbers: \(x = -3, x = 9\)
Test intervals: \((-\infty, -3), (-3, 9), (9, \infty)\)
Test: Is \((x + 3)(x - 9) < 0?\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-3, 9)\)

134. \(x^2 - 2x \geq 3\)
\(x^2 - 2x - 3 \geq 0\)
Critical numbers: \(x = -1, x = 3\)
Test intervals:
\((-\infty, -1) \Rightarrow (x - 3)(x + 1) > 0\)
\((-1, 3) \Rightarrow (x - 3)(x + 1) < 0\)
\((3, \infty) \Rightarrow (x - 3)(x + 1) > 0\)
Solution interval: \((-\infty, -1) \cup [3, \infty)\)
135. \[6x^2 + 5x < 4\]
\[6x^2 + 5x - 4 < 0\]
\[(3x + 4)(2x - 1) < 0\]
Critical numbers: \(x = -\frac{4}{3}, x = \frac{1}{2}\)
Test intervals: \((-\infty, -\frac{4}{3}), (-\frac{4}{3}, \frac{1}{2}), (\frac{1}{2}, \infty)\)
Test: Is \((3x + 4)(2x - 1) < 0?\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-\frac{4}{3}, \frac{1}{2})\)

137. \[x^3 - 16x \geq 0\]
\[x(x + 4)(x - 4) \geq 0\]
Critical numbers: \(x = 0, x = \pm 4\)
Test intervals: \((-\infty, -4), (-4, 0), (0, 4), (4, \infty)\)
Test: Is \(x(x + 4)(x - 4) \geq 0?\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \([-4, 0] \cup [4, \infty)\)

139. \[\frac{x + 8}{x - 3} - 2 < 0\]
\[\frac{x + 8 - 2(x + 5)}{x - 3} < 0\]
\[\frac{-x - 2}{x - 3} < 0\]
Critical numbers: \(x = -2, x = -5\)
Test intervals: \((-\infty, -5), (-5, -2), (-2, \infty)\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-\infty, -5) \cup (-2, \infty)\)

141. \[\frac{2}{x + 1} \leq \frac{3}{x - 1}\]
\[\frac{2(x - 1) - 3(x + 1)}{(x + 1)(x - 1)} \leq 0\]
\[\frac{2x - 2 - 3x - 3}{(x + 1)(x - 1)} \leq 0\]
\[\frac{-x - 5}{(x + 1)(x - 1)} \leq 0\]
Critical numbers: \(x = -5, x = \pm 1\)
Test intervals: \((-\infty, -5), (-5, -1), (-1, 1), (1, \infty)\)
Test: Is \(-x - 5 \leq 0?\)
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \([-5, -1] \cup (1, \infty)\)

136. \[2x^2 + x \geq 15\]
\[2x^2 + x - 15 \geq 0\]
\[(2x - 5)(x + 3) \geq 0\]
Critical numbers: \(x = \frac{5}{2}, x = -3\)
Test intervals: \((-\infty, -3) \Rightarrow (2x - 5)(x + 3) > 0\)
\[(-3, \frac{5}{2}) \Rightarrow (2x - 5)(x + 3) < 0\]
\[(\frac{5}{2}, \infty) \Rightarrow (2x - 5)(x + 3) > 0\]
Solution interval: \((-\infty, -3] \cup [\frac{5}{2}, \infty)\)

138. \[12x^3 - 20x^2 < 0\]
\[4x^2(3x - 5) < 0\]
Critical numbers: \(x = 0, x = \frac{5}{3}\)
Test intervals: \((-\infty, 0) \Rightarrow 12x^3 - 20x^2 < 0\)
\[(0, \frac{5}{3}) \Rightarrow 12x^3 - 20x^2 < 0\]
\[(\frac{5}{3}, \infty) \Rightarrow 12x^3 - 20x^2 > 0\]
Solution interval: \((-\infty, 0) \cup (0, \frac{5}{3})\)

140. \[3x + 8 \leq 4x - 12\]
\[3x + 8 \leq 4x - 12\]
\[x \geq 20\]
Critical numbers: \(x = 3, x = 20\)
Test intervals: \((-\infty, 3) \Rightarrow \frac{3x + 8}{x - 3} < 4\)
\[(3, 20) \Rightarrow \frac{3x + 8}{x - 3} \geq 4\]
\[(20, \infty) \Rightarrow \frac{3x + 8}{x - 3} < 4\]
Solution interval: \((-\infty, 3) \cup (20, \infty)\)

142. \[\frac{x - 5}{3 - x} < 0\]
Critical numbers: \(x = 5, x = 3\)
Test intervals: \((-\infty, 3) \Rightarrow \frac{x - 5}{3 - x} < 0\)
\[(3, 5) \Rightarrow \frac{x - 5}{3 - x} > 0\]
\[(5, \infty) \Rightarrow \frac{x - 5}{3 - x} < 0\]
Solution intervals: \((-\infty, 3) \cup (5, \infty)\)
143. \[
\frac{x^2 + 7x + 12}{x} \geq 0
\]
\[
\frac{(x + 4)(x + 3)}{x} \geq 0
\]
Critical numbers: \(x = -4, x = -3, x = 0\)
Test intervals: \((-\infty, -4), (-4, -3), (-3, 0), (0, \infty)\)
Test: Is \(\frac{(x + 4)(x + 3)}{x} \geq 0\)?

By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \([-4, -3] \cup (0, \infty)\)

144. \[
\frac{1}{x - 2} > \frac{1}{x}
\]
\[
\frac{1}{x - 2} - \frac{1}{x} > 0
\]
Critical numbers: \(x = 2, x = 0\)
Test intervals: \((-\infty, 0) \Rightarrow \frac{1}{x - 2} - \frac{1}{x} > 0\)
\((0, 2) \Rightarrow \frac{1}{x - 2} - \frac{1}{x} < 0\)
\((2, \infty) \Rightarrow \frac{1}{x - 2} - \frac{1}{x} > 0\)
Solution interval: \((-\infty, 0) \cup (2, \infty)\)

145. \[
5000(1 + r)^2 > 5500
\]
\[
(1 + r)^2 > 1.1
\]
\[
1 + r > 1.0488
\]
\[
r > 0.0488
\]
\[
r > 4.9\%
\]

146. \[
P = \frac{1000(1 + 3r)}{5 + t}
\]
\[
2000 \leq \frac{1000(1 + 3r)}{5 + t}
\]
\[
2000(5 + t) \leq 1000(1 + 3t)
\]
\[
10,000 + 2000t \leq 1000 + 3000t
\]
\[
-1000 \leq -9000
\]
\[
t \geq 9 \text{ days}
\]

147. False
\[
\sqrt{-18} \sqrt{-2} = (\sqrt{18i})(\sqrt{2i}) = \sqrt{36i^2} = -6
\]
\[
\sqrt{(-18)(-2)} = \sqrt{36} = 6
\]

148. False. The equation has no real solution.
The solutions are
\[
\frac{717}{650} \pm \frac{\sqrt{33111}i}{650}
\]

149. Rational equations, equations involving radicals, and absolute value equations, may have “solutions” that are extraneous. So checking solutions, in the original equations, is crucial to eliminate these extraneous values.

150. Sample answer: The first equivalent inequality written is incorrect. It should be \(11x + 4 \leq -26\). This leads to the solution \((-\infty, -\frac{26}{11}] \cup [2, \infty)\).

**Problem Solving for Chapter 1**

1. [Diagram of distance-time graph]
2. (a) $1 + 2 + 3 + 4 + 5 = 15$
    $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$
    $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

(b) $1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}$
    When $n = 5$: $\frac{1}{2}(5)(6) = 15$
    When $n = 8$: $\frac{1}{2}(8)(9) = 36$
    When $n = 10$: $\frac{1}{2}(10)(11) = 55$

3. (a) $A = \pi ab$  
    $a + b = 20 \implies b = 20 - a$, thus:
    $A = \pi a(20 - a)$

(b) \[
\begin{array}{c|cccc}
   a  & 4  & 7  & 10 & 13 \\
   A  & 64\pi & 91\pi & 100\pi & 91\pi & 64\pi \\
\end{array}
\]

(c) $300 = \pi a(20 - a)$
    $300 = 20\pi a - \pi a^2$
    $\pi a^2 - 20\pi a + 300 = 0$
    $a = \frac{20\pi \pm \sqrt{(-20\pi)^2 - 4\pi(300)}}{2\pi}$
    $= \frac{20\pi \pm 20\sqrt{\pi(\pi - 3)}}{2\pi}$
    $= 10 \pm \frac{10}{\pi}\sqrt{\pi(\pi - 3)}$
    $a = 12.123$ or $a = 7.877$

4. $P = 0.00256a^2$
   (a) $0.00256a^2 = 20$
       $a^2 = 7812.5$
       $a = 88.4$ miles per hour.
   (b) $0.00256a^2 = 40$
       $a^2 = 15625$
       $a = 125$ miles per hour.

No, actually it can survive wind blowing at $\sqrt{2}$ times the speed found in part (a).

(c) The wind speed in the formula is squared, so a small increase in wind speed could have potentially serious effects on a building.

5. $h = \left( \sqrt{h_0} - \frac{2\pi d^2 \sqrt{3}}{lw} t \right)^2$
   $l = 60^\circ, w = 30^\circ, h_0 = 25^\circ, d = 2^\circ$
   $h = \left( 5 - \frac{8\pi \sqrt{3}}{1800} t \right)^2 = \left( 5 - \frac{\pi \sqrt{3}}{225} t \right)^2$

(a) $12.5 = \left( 5 - \frac{\pi \sqrt{3}}{225} t \right)^2$
    $\sqrt{12.5} = 5 - \frac{\pi \sqrt{3}}{225} t$
    $t = \frac{225}{\pi \sqrt{3}} (5 - \sqrt{12.5}) = 60.6$ seconds

(b) $0 = \left( \sqrt{12.5} - \frac{\pi \sqrt{3}}{225} t \right)^2$
    $t = \frac{225 \sqrt{12.5}}{\pi \sqrt{3}} = 146.2$ seconds

(c) The speed at which the water drains decreases as the amount of the water in the bathtub decreases.
6. (a) If \( x^2 + 9 = (x + m)(x + n) \) then 
   \[ mn = 9 \text{ and } m + n = 0. \]

   (b) \( m + n = 0 \implies n = -m \)
   \[ m(-m) = 9 \implies -m^2 = 9 \implies m^2 = -9 \]
   There is no integer \( m \) such that \( m^2 \) equals a negative number. \( x^2 + 9 \) cannot be factored over the integers.

7. (a) 5, 12, and 13; 8, 15, and 17

   (b) \( 5 \cdot 12 \cdot 13 = 780 \) which is divisible by 3, 4, and 5
   \( 8 \cdot 15 \cdot 17 = 2040 \) which is divisible by 3, 4, and 5
   \( 7 \cdot 24 \cdot 25 = 4200 \) which is also divisible by 3, 4, and 5

   (c) Conjecture: If \( a^2 + b^2 = c^2 \) where \( a, b, \) and \( c \) are positive integers, then \( abc \) is divisible by 60.

8. | Equation | \( x_1, x_2 \) | \( x_1 + x_2 \) | \( x_1 \cdot x_2 \) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( x^2 - x - 6 = 0 ) ( (x + 2)(x - 3) = 0 )</td>
<td>-2, 3</td>
<td>1</td>
<td>-6</td>
</tr>
<tr>
<td>(b) ( 2x^2 + 5x - 3 = 0 ) ( (2x - 1)(x + 3) = 0 )</td>
<td>( \frac{1}{2}, -3 )</td>
<td>( -\frac{5}{2} )</td>
<td>( -\frac{3}{2} )</td>
</tr>
<tr>
<td>(c) ( 4x^2 - 9 = 0 ) ( (2x + 3)(2x - 3) = 0 )</td>
<td>( -\frac{3}{2}, \frac{3}{2} )</td>
<td>0</td>
<td>( -\frac{9}{4} )</td>
</tr>
<tr>
<td>(d) ( x^2 - 10x + 34 = 0 ) ( x = 5 \pm 3i )</td>
<td>5 + 3i, 5 - 3i</td>
<td>10</td>
<td>34</td>
</tr>
</tbody>
</table>

9. \( ax^2 + bx + c = 0 \)

   \[ x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \]
   \[ x_1 + x_2 = \frac{b}{a} \]
   \[ x_1 \cdot x_2 = \frac{c}{a} \]

10. (a) (i) \( \left( \frac{-5 + 5\sqrt{3}i}{2} \right)^3 = 125 \)
    (ii) \( \left( \frac{-5 - 5\sqrt{3}i}{2} \right)^3 = 125 \)

    (b) (i) \( \left( \frac{-3 + 3\sqrt{3}i}{2} \right)^3 = 27 \)
    (ii) \( \left( \frac{-3 - 3\sqrt{3}i}{2} \right)^3 = 27 \)

    (c) (i) The cube roots of 1 are:
    \[ 1, -\frac{1 \pm \sqrt{3}i}{2} \]
    (ii) The cube roots of 8 are:
    \[ 2, -1 \pm \sqrt{3}i \]
    (iii) The cube roots of 64 are:
    \[ 4, -2 \pm 2\sqrt{3}i \]

11. (a) \( z_m = \frac{1}{z} \)
    \[ = \frac{1}{1 + i} = \frac{1}{1} \cdot \frac{1 - i}{1 - i} = \frac{1 - i}{2} = \frac{1}{2} - \frac{1}{2}i \]

    (b) \( z_m = \frac{1}{z} \)
    \[ = \frac{1}{3 - i} = \frac{1}{3} \cdot \frac{3 + i}{3 + i} = \frac{3 + i}{10} = \frac{3}{10} + \frac{1}{10}i \]

    (c) \( z_m = \frac{1}{z} \)
    \[ = \frac{1}{-2 + 8i} = \frac{1}{-2 + 8i} \cdot \frac{-2 - 8i}{-2 - 8i} = \frac{-2 - 8i}{68} = \frac{1}{34} - \frac{2}{17}i \]

12. \( (a + bi)(a - bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2 \)
   Since \( a \) and \( b \) are real numbers, \( a^2 + b^2 \) is also a real number.
13. (a) $c = i$

The terms are: $i, -1 + i, -i, -1 + i, -i, -1 + i, -i, -1 + i, -i, \ldots$

The sequence is bounded so $c = i$ is in the Mandelbrot Set.

(b) $c = 1 + i$

The terms are: $1 + i, 1 + 3i, -7 + 7i, 1 - 97i, -9407 - 193i, \ldots$

The sequence is unbounded so $c = 1 + i$ is not in the Mandelbrot Set.

(c) $c = -2$

The terms are: $-2, 2, 2, 2, \ldots$

The sequence is bounded so $c = -2$ is in the Mandelbrot Set.

14. \[ 4\sqrt{x} = 2x + k \]

\[ 2x - 4\sqrt{x} + k = 0 \quad \text{Complete the square.} \]

\[ x - 2\sqrt{x} = \frac{-k}{2} \]

\[ x - 2\sqrt{x} + 1 = 1 - \frac{k}{2} \]

\[ (\sqrt{x} - 1)^2 = 1 - \frac{k}{2} \]

This equation will have two solutions when $1 - \frac{k}{2} > 0$ or when $k < 2$.

This equation will have one solution when $1 - \frac{k}{2} = 0$ or when $k = 2$.

This equation will have no solutions when $1 - \frac{k}{2} < 0$ or when $k > 2$.

15. \[ y = x^4 - x^3 - 6x^2 + 4x + 8 \]

\[ = (x - 2)^2(x + 1)(x + 2) \]

From the graph we see that $x^4 - x^3 - 6x^2 + 4x + 8 > 0$

on the intervals $(-\infty, -2) \cup (-1, 2) \cup (2, \infty)$.

16. \[ |w - 16| \leq \frac{1}{2} \]

\[-\frac{1}{2} \leq w - 16 \leq \frac{1}{2} \]

\[ 15.5 \leq w \leq 16.5 \]

\[ 4(15.5) = 62, 4(16.5) = 66 \]

The greatest amount you could expect to get is 66 ounces and the least amount is 62 ounces.
Practice Test for Chapter 1

1. Graph $3x - 5y = 15.$
   2. Graph $y = \sqrt{9 - x}.$

3. Solve $5x + 4 = 7x - 8.$
   4. Solve $\frac{x}{3} - 5 = \frac{x}{5} + 1.$

5. Solve $\frac{3x + 1}{6x - 7} = \frac{2}{5}.$
   6. Solve $(x - 3)^2 + 4 = (x + 1)^2.$

7. Solve $A = \frac{1}{2}(a + b)h$ for $a.$
   8. 301 is what percent of 4300?

9. Cindy has $6.05 in quarters and nickels. How many of each coin does she have if there are 53 coins in all?

10. Ed has $15,000 invested in two funds paying 9\frac{1}{2}\%$ and 11% simple interest, respectively. How much is invested in each if the yearly interest is $1582.50? 11. Solve $28 + 5x - 3x^2 = 0$ by factoring.

12. Solve $(x - 2)^2 = 24$ by taking the square root of both sides.

13. Solve $x^2 - 4x - 9 = 0$ by completing the square.

14. Solve $x^2 + 5x - 1 = 0$ by the Quadratic Formula.

15. Solve $3x^2 - 2x + 4 = 0$ by the Quadratic Formula.

16. The perimeter of a rectangle is 1100 feet. Find the dimensions so that the enclosed area will be 60,000 square feet.

17. Find two consecutive even positive integers whose product is 624.

18. Solve $x^3 - 10x^2 + 24x = 0$ by factoring.
   19. Solve $\sqrt[5]{6 - x} = 4.$

20. Solve $(x^2 - 8)^{2/3} = 4.$
   21. Solve $x^4 - x^2 - 12 = 0.$

22. Solve $4 - 3x > 16.$
   23. Solve $\left|\frac{x - 3}{2}\right| < 5.$

24. Solve $\frac{x + 1}{x - 3} < 2.$
   25. Solve $|3x - 4| \geq 9.$