1-1 Functions

Write each set of numbers in set-builder and interval notation, if possible.

1. \( x > 50 \)

**SOLUTION:**
The set includes all real numbers greater than 50. In set-builder notation this set can be described as \( \{ x \mid x > 50, x \in \mathbb{R} \} \), and in interval notation it can be described as \((50, \infty)\).

**ANSWER:**
\( \{ x \mid x > 50, x \in \mathbb{R} \}; (50, \infty) \)

2. \( x < -13 \)

**SOLUTION:**
The set includes all real numbers less than \(-13\). In set-builder notation this set can be described as \( \{ x \mid x < -13, x \in \mathbb{R} \} \), and in interval notation it can be described as \((-\infty, -13)\).

**ANSWER:**
\( \{ x \mid x < -13, x \in \mathbb{R} \}; (-\infty, -13) \)

3. \( x \leq -4 \)

**SOLUTION:**
The set includes all real numbers less than or equal to \(-4\). In set-builder notation this set can be described as \( \{ x \mid x \leq -4, x \in \mathbb{R} \} \), and in interval notation it can be described as \((-\infty, -4]\).

**ANSWER:**
\( \{ x \mid x \leq -4, x \in \mathbb{R} \}; (-\infty, -4] \)

4. \( -4, -3, -2, -1, \ldots \)

**SOLUTION:**
The set includes all integers greater than or equal to \(-4\). In set-builder notation this set can be described as \( \{ x \mid x \geq -4, x \in \mathbb{Z} \} \). The set cannot be described using one or more inequalities, and therefore cannot be written in interval notation.

**ANSWER:**
\( \{ x \mid -4 \leq x, x \in \mathbb{Z} \} \)

5. \( 8 < x < 99 \)

**SOLUTION:**
The set includes all real numbers greater than 8 and less than 99. In set-builder notation this set can be described as \( \{ x \mid 8 < x < 99, x \in \mathbb{R} \} \), and in interval notation it can be described as \((8, 99)\).

**ANSWER:**
\( \{ x \mid 8 < x < 99, x \in \mathbb{R} \}; (8, 99) \)

6. \(-31 < x \leq 64 \)

**SOLUTION:**
The set includes all real numbers greater than \(-31\) and less than or equal to 64. In set-builder notation this set can be described as \( \{ x \mid -31 < x \leq 64, x \in \mathbb{R} \} \), and in interval notation it can be described as \((-31, 64]\).

**ANSWER:**
\( \{ x \mid -31 < x \leq 64, x \in \mathbb{R} \}; (-31, 64] \)

7. \( x < -19 \text{ or } x > 21 \)

**SOLUTION:**
The set includes all real numbers less than \(-19\) or greater than 21. In set-builder notation this set can be described as \( \{ x \mid x < -19 \text{ or } x > 21, x \in \mathbb{R} \} \), and in interval notation it can be described as \((-\infty, -19) \cup (21, \infty)\).

**ANSWER:**
\( \{ x \mid x < -19 \text{ or } x > 21, x \in \mathbb{R} \}; (-\infty, -19) \cup (21, \infty) \)

8. \( x < 0 \text{ or } x \geq 100 \)

**SOLUTION:**
The set includes all real numbers less than 0 or greater than or equal to 100. In set-builder notation this set can be described as \( \{ x \mid x < 0 \text{ or } x \geq 100, x \in \mathbb{R} \} \), and in interval notation it can be described as \((-\infty, 0) \cup [100, \infty)\).

**ANSWER:**
\( \{ x \mid x < 0 \text{ or } x \geq 100, x \in \mathbb{R} \}; (-\infty, 0) \cup [100, \infty) \)
1-1 Functions

9. \( \{-0.25, 0, 0.25, 0.50, \ldots\}\)

**SOLUTION:**
The set includes multiples of 0.25, starting with 0.25 \((-1)\) or \(-0.25\). Therefore, in set-builder notation the set can be described as \( \{ x \mid 0.25n = x, n \geq -1, n \in \mathbb{Z}\}\). The set cannot be described using one or more inequalities, and therefore cannot be written in interval notation.

**ANSWER:**
\( \{ x \mid 0.25n = x, n \geq -1, n \in \mathbb{Z}\}\)

10. \( x \leq 61 \) or \( x \geq 67 \)

**SOLUTION:**
This set includes all real numbers that are less than or equal to 61 and greater than or equal to 67. Therefore, in set-builder notation the set can be described as \( \{ x \mid x \leq 61 \text{ or } x \geq 67, x \in \mathbb{R}\}\), and in interval notation it can be described as \((-\infty, 61] \cup [67, \infty)\).

**ANSWER:**
\( \{ x \mid x \leq 61 \text{ or } x \geq 67, x \in \mathbb{R}\}; (-\infty, 61] \cup [67, \infty)\)

11. \( x \leq -45 \) or \( x > 86 \)

**SOLUTION:**
This set includes all real numbers that are less than or equal to \(-45\) and greater than 86. Therefore, in set-builder notation the set can be described as \( \{ x \mid x \leq -45 \text{ or } x > 86, x \in \mathbb{R}\}\), and in interval notation it can be described as \((-\infty, -45] \cup (86, \infty)\).

**ANSWER:**
\( \{ x \mid x \leq -45 \text{ or } x > 86, x \in \mathbb{R}\}; (-\infty, -45] \cup (86, \infty)\)

12. all multiples of 8

**SOLUTION:**
This set includes all integers that are multiples of 8. Therefore, in set-builder notation the set can be described as \( \{ x \mid x = 8n, n \in \mathbb{Z}\}\). The set cannot be described using one or more inequalities, and therefore cannot be written in interval notation.

**ANSWER:**
\( \{ x \mid x = 8n, n \in \mathbb{Z}\}\)

13. all multiples of 5

**SOLUTION:**
This set includes all integers that are multiples of 5. Therefore, in set-builder notation the set can be described as \( \{ x \mid x = 5n, n \in \mathbb{Z}\}\). The set cannot be described using one or more inequalities, and therefore cannot be written in interval notation.

**ANSWER:**
\( \{ x \mid x = 5n, n \in \mathbb{Z}\}\)

14. \( x \geq 32 \)

**SOLUTION:**
The set includes all real numbers greater than or equal to 32. Therefore, in set-builder notation the set can be described as \( \{ x \mid x \geq 32, x \in \mathbb{R}\}\), and in interval notation the set can be described as \([32, \infty)\).

**ANSWER:**
\( \{ x \mid x \geq 32, x \in \mathbb{R}\}; [32, \infty)\)

**Determine whether each relation represents \( y \) as a function of \( x \).**

15. The input value \( x \) is a bank account number and the output value \( y \) is the account balance.

**SOLUTION:**
Each value of \( x \) cannot be assigned to more than one \( y \)-value because a bank account can only have one balance at a given time. Therefore, the sentence describes \( y \) as a function of \( x \).

**ANSWER:**
function

16. The input value \( x \) is the year and the output value \( y \) is the day of the week.

**SOLUTION:**
Each value of \( x \) can be assigned to more than one \( y \)-value because a given year corresponds to more than one day of the week. Therefore, the sentence does not describe \( y \) as a function of \( x \).

**ANSWER:**
not a function
1-1 Functions

17.  \( x \)   \( y \)
\[ \begin{array}{c|c}
-50 & 2.11 \\
-40 & 2.14 \\
-30 & 2.16 \\
-20 & 2.17 \\
-10 & 2.17 \\
  0 & 2.18 \\
\end{array} \]

**SOLUTION:**
Each \( x \)-value is assigned to exactly one \( y \)-value. Therefore, the table represents \( y \) as a function of \( x \).

**ANSWER:**
function

20. \( x^2 = y + 2 \)

**SOLUTION:**
To determine whether this equation represents \( y \) as a function of \( x \), solve the equation for \( y \).

\[
x^2 = y + 2 \\
x^2 - 2 = y
\]

This equation represents \( y \) as a function of \( x \), because for every \( x \)-value there is exactly one corresponding \( y \)-value.

**ANSWER:**
function

21. \( 3y + 4x = 11 \)

**SOLUTION:**
To determine whether this equation represents \( y \) as a function of \( x \), solve the equation for \( y \).

\[
3y + 4x = 11 \\
3y = 11 - 4x \\
y = \frac{11 - 4x}{3}
\]

This equation represents \( y \) as a function of \( x \), because for every \( x \)-value there is exactly one corresponding \( y \)-value.

**ANSWER:**
function

22. \( 4y^2 + 18 = 96x \)

**SOLUTION:**
To determine whether this equation represents \( y \) as a function of \( x \), solve the equation for \( y \).

\[
4y^2 + 18 = 96x \\
4y^2 = 96x - 18 \\
y^2 = \frac{96x - 18}{4} \\
y^2 = \frac{48x - 9}{2} \\
y = \pm \sqrt{\frac{48x - 9}{2}}
\]

This equation does not represent \( y \) as a function of \( x \) because there will be two corresponding \( y \)-values, one positive and one negative, for any \( x \)-value greater than 0.1875.

**ANSWER:**
not a function
1-1 Functions

23. \( \sqrt{48}y = x \)

\textit{SOLUTION:}
To determine whether this equation represents \( y \) as a function of \( x \), solve the equation for \( y \).
\[
\sqrt{48}y = x \\
\sqrt{48}y = \sqrt{48}x \\
y = \frac{x}{\sqrt{48}} \\
y = \frac{48}{48}
\]
This equation represents \( y \) as a function of \( x \), because for every \( x \)-value there is exactly one corresponding \( y \)-value.

\textit{ANSWER:}
function

24. \( \frac{x}{y} = y - 6 \)

\textit{SOLUTION:}
To determine whether this equation represents \( y \) as a function of \( x \), solve the equation for \( y \).
\[
\frac{x}{y} = y - 6 \\
x = y^2 - 6y \\
This equation does not represent \( y \) as a function of \( x \), because for every \( x \)-value there will be two corresponding \( y \)-values.

\textit{ANSWER:}
not a function

25. \[ \text{Diagram of a graph passing the vertical line test.} \]

\textit{SOLUTION:}
The graph passes the vertical line test. Therefore, the graph represents \( y \) as a function of \( x \).

\textit{ANSWER:}
function

26. \[ \text{Diagram of a circle.} \]

\textit{SOLUTION:}
A vertical line at \( x = 1 \) intersects the graph at more than one point. Therefore, the graph does not represent \( y \) as a function of \( x \).

\textit{ANSWER:}
not a function
29. **METEOROLOGY** The five-day forecast for a city is shown.

![Temperature Forecast]

**SOLUTION:**

- **a.** Represent the relation between the day of the week and the estimated high temperature as a set of ordered pairs.
- **b.** Is the estimated high temperature a function of the day of the week? the low temperature? Explain your reasoning.

**SOLUTION:**

- **a.** Let set $A$ represent the days of the week and set $B$ represent the high temperatures. Therefore, set $A = \{1, 2, 3, 4, 5\}$ and set $B = \{70, 75, 70, 62, 65\}$, and the set of ordered pairs that represents the relation from set $A$ to set $B$ is $\{(1, 70), (2, 75), (3, 70), (4, 62), (5, 65)\}$.
- **b.** The estimated high temperature is a function of the day of the week because there is exactly one estimated high temperature each day. The estimated low temperature is also a function of the day of the week because there is exactly one estimated low temperature each day.

**ANSWER:**

- **a.** $\{(1, 70), (2, 75), (3, 70), (4, 62), (5, 65)\}$
- **b.** Yes; there is exactly one estimated high temperature each day. Yes; there is exactly one low temperature each day.
1-1 Functions

Find each function value.

30. \( g(x) = 2x^2 + 18x - 14 \)
   a. \( g(9) \)
   b. \( g(3x) \)
   c. \( g(1 + 5m) \)

**SOLUTION:**

To find \( g(9) \), replace \( x \) with 9 in \( g(x) = 2x^2 + 18x - 14 \).

\[
g(x) = 2x^2 + 18x - 14 \\
g(9) = 2(9)^2 + 18(9) - 14 \\
     = 162 + 162 - 14 \\
     = 310
\]

To find \( g(3x) \), replace \( x \) with 3 in \( g(x) = 2x^2 + 18x - 14 \).

\[
g(x) = 2x^2 + 18x - 14 \\
g(3x) = 2(3x)^2 + 18(3x) - 14 \\
     = 18x^2 + 54x - 14
\]

To find \( g(1 + 5m) \), replace \( x \) with 1 + 5m in \( g(x) = 2x^2 + 18x - 14 \).

\[
g(x) = 2x^2 + 18x - 14 \\
g(1 + 5m) = 2(1 + 5m)^2 + 18(1 + 5m) - 14 \\
     = 50m^2 + 110m + 6
\]

**ANSWER:**

a. 310
b. \( 18x^2 + 54x - 14 \)
c. \( 50m^2 + 110m + 6 \)

31. \( h(y) = -3y^3 - 6y + 9 \)
   a. \( h(4) \)
   b. \( h(-2y) \)
   c. \( h(5b + 3) \)

**SOLUTION:**

To find \( h(4) \), replace \( y \) with 4 in \( h(y) = -3y^3 - 6y + 9 \).

\[
h(y) = -3y^3 - 6y + 9 \\
h(4) = -3(4)^3 - 6(4) + 9 \\
     = -192 - 24 + 9 \\
     = -207
\]

To find \( h(-2y) \), replace \( y \) with \(-2y\) in \( h(y) = -3y^3 - 6y + 9 \).

\[
h(y) = -3y^3 - 6y + 9 \\
h(-2y) = -3(-2y)^3 - 6(-2y) + 9 \\
     = 24y^3 + 12y + 9
\]

To find \( h(5b + 3) \), replace \( y \) with 5b + 3 in \( h(y) = -3y^3 - 6y + 9 \).

\[
h(y) = -3y^3 - 6y + 9 \\
h(5b + 3) = -3(5b + 3)^3 - 6(5b + 3) + 9 \\
     = 375b^3 - 675b^2 - 405b - 81 - 30b - 18 + 9 \\
     = 375b^3 - 675b^2 - 435b - 90
\]

**ANSWER:**

a. -207
b. \( 24y^3 + 12y + 9 \)
c. \( -375b^3 - 675b^2 - 435b - 90 \)
1-1 Functions

32. \( f(t) = \frac{4t+11}{3t^2+5t+1} \)
   a. \( f(-6) \)
   b. \( f(4t) \)
   c. \( f(3-2a) \)

   **SOLUTION:**
   To find \( f(-6) \), replace \( t \) with \(-6\) in \( f(t) = \frac{4t+11}{3t^2+5t+1} \).
   \[ f(t) = \frac{4t+11}{3t^2+5t+1} \]
   \[ f(-6) = \frac{4(-6)+11}{3(-6)^2+5(-6)+1} \]
   \[ = \frac{-24+11}{108-30+1} \]
   \[ = \frac{-13}{79} \]

   To find \( f(4t) \), replace \( t \) with \( 4t \) in \( f(t) = \frac{4t+11}{3t^2+5t+1} \).
   \[ f(t) = \frac{4t+11}{3t^2+5t+1} \]
   \[ f(4t) = \frac{4(4t)+11}{3(4t)^2+5(4t)+1} \]
   \[ = \frac{16t+11}{48t^2+20t+1} \]

   To find \( f(3-2a) \), replace \( t \) with \( 3-2a \) in \( f(t) = \frac{4t+11}{3t^2+5t+1} \).
   \[ f(t) = \frac{4t+11}{3t^2+5t+1} \]
   \[ f(3-2a) = \frac{4(3-2a)+11}{3(3-2a)^2+5(3-2a)+1} \]
   \[ = \frac{12-8a+11}{27-36a+12a^2+15-10a+1} \]
   \[ = \frac{-8a+23}{12a^2-46a+43} \]

   **ANSWER:**
   a. \( -\frac{13}{79} \)
   b. \( \frac{16t+11}{48t^2+20t+1} \)
   c. \( \frac{12a^2-46a+43}{8a+23} \)

33. \( g(x) = \frac{3x^3}{x^2+x-4} \)
   a. \( g(-2) \)
   b. \( g(5x) \)
   c. \( g(8-4b) \)

   **SOLUTION:**
   To find \( g(-2) \), replace \( x \) with \(-2\) in \( g(x) = \frac{3x^3}{x^2+x-4} \).
   \[ g(x) = \frac{3x^3}{x^2+x-4} \]
   \[ g(-2) = \frac{3(-2)^3}{(-2)^2+(-2)-4} \]
   \[ = \frac{-24}{-24} \]
   \[ = \frac{-24}{-2} \]
   \[ = -12 \]

   To find \( g(5x) \), replace \( x \) with \( 5x \) in \( g(x) = \frac{3x^3}{x^2+x-4} \).
   \[ g(x) = \frac{3x^3}{x^2+x-4} \]
   \[ g(5x) = \frac{3(5x)^3}{(5x)^2+(5x)-4} \]
   \[ = \frac{375x^3}{25x^2+5x-4} \]

   To find \( g(8-4b) \), replace \( x \) with \( 8-4b \) in \( g(x) = \frac{3x^3}{x^2+x-4} \).
   \[ g(x) = \frac{3x^3}{x^2+x-4} \]
   \[ g(8-4b) = \frac{3(8-4b)^3}{(8-4b)^2+(8-4b)-4} \]
   \[ = \frac{\frac{375x^3}{25x^2+5x-4}}{\frac{16b^2-64b+64+8-4b-4}{4b^2-17b+17}} \]
   \[ = \frac{375x^3}{25x^2+5x-4-48b^3+288b^2-576b+384}{4b^2-17b+17} \]

   **ANSWER:**
   a. \( 12 \)
   b. \( \frac{375x^3}{25x^2+5x-4} \)
   c. \( \frac{375x^3}{25x^2+5x-4-48b^3+288b^2-576b+384} \)
34. \( h(x) = 16 - \frac{12}{2x+3} \)

a. \( h(-3) \)
b. \( h(6x) \)
c. \( h(10-2c) \)

**SOLUTION:**
To find \( h(-3) \), replace \( x \) with \(-3\) in \( h(x) = 16 - \frac{12}{2x+3} \).

\[
h(-3) = 16 - \frac{12}{2(-3)+3} = 16 - \frac{12}{-6+3} = 16 - \frac{12}{-3} = 16 + 4 \text{ or } 20
\]

To find \( h(6x) \), replace \( x \) with \( 6x \) in \( h(x) = 16 - \frac{12}{2x+3} \).

\[
h(6x) = 16 - \frac{12}{2(6x)+3} = 16 - \frac{12}{12x+3} = 16 - \frac{4}{4x+1}
\]

To find \( h(10-2c) \), replace \( x \) with \( 10-2c \) in \( h(x) = 16 - \frac{12}{2x+3} \).

\[
h(10-2c) = 16 - \frac{12}{2(10-2c)+3} = 16 - \frac{12}{20-4c+3} = 16 - \frac{12}{17-4c}
\]

**ANSWER:**

a. 20

b. \( 16 - \frac{4}{4x+1} \)

c. \( 16 - \frac{12}{23-4c} \)

35. \( f(x) = -7 + \frac{6x+1}{x} \)

a. \( f(5) \)
b. \( f(-8x) \)
c. \( f(6y + 4) \)

**SOLUTION:**
To find \( f(5) \), replace \( x \) with 5 in \( f(x) = -7 + \frac{6x+1}{x} \).

\[
f(5) = -7 + \frac{6(5)+1}{5} = -7 + \frac{31}{5} = -7 + 6.2 = -0.8
\]

To find \( f(-8x) \), replace \( x \) with \(-8x\) in \( f(x) = -7 + \frac{6x+1}{x} \).

\[
f(-8x) = -7 + \frac{6(-8x)+1}{-8x} = -7 - \frac{48x+1}{8x} = -7 - 6 - \frac{1}{8x} = -1 - \frac{1}{8x}
\]

To find \( f(6y + 4) \), replace \( x \) with \( 6y + 4 \) in \( f(x) = -7 + \frac{6x+1}{x} \).

\[
f(6y+4) = -7 + \frac{6(6y+4)+1}{6y+4} = -7 + \frac{36y+25}{6y+4}
\]
1-1 Functions

**ANSWER:**

a. $-0.8$

b. $-1 - \frac{1}{8x}$

c. $-7 + \frac{36y + 25}{6y + 4}$

36. $g(m) = 3 + \sqrt{m^2 - 4}$

a. $g(-2)$

b. $g(3m)$

c. $g(4m - 2)$

**SOLUTION:**

To find $g(-2)$, replace $x$ with $-2$ in $g(m) = 3 + \sqrt{m^2 - 4}$.

$g(x) = 3 + \sqrt{m^2 - 4}$

$g(-2) = 3 + \sqrt{(-2)^2 - 4}$

$= 3 + \sqrt{4 - 4}$

$= 3$

To find $g(3m)$, replace $x$ with $3m$ in $g(m) = 3 + \sqrt{m^2 - 4}$.

$g(x) = 3 + \sqrt{m^2 - 4}$

$g(3m) = 3 + \sqrt{(3m)^2 - 4}$

$= 3 + \sqrt{9m^2 - 4}$

To find $g(4m - 2)$, replace $x$ with $4m - 2$ in $g(m) = 3 + \sqrt{m^2 - 4}$.

$g(x) = 3 + \sqrt{m^2 - 4}$

$g(-2) = 3 + \sqrt{(4m - 2)^2 - 4}$

$= 3 + \sqrt{16m^2 - 16m + 4 - 4}$

$= 3 + \sqrt{16m^2 - 16m}$

$= 3 + 4\sqrt{m^2 - m}$

37. $t(x) = 5\sqrt{6x^2}$

a. $t(-4)$

b. $t(2x)$

c. $t(7 + n)$

**SOLUTION:**

To find $t(-4)$, replace $x$ with $-4$ in $t(x) = 5\sqrt{6x^2}$.

$t(x) = 5\sqrt{6x^2}$

t$(-4) = 5\sqrt{6(-4)^2}$

$= 5\sqrt{96}$

$= 20\sqrt{6}$

To find $t(2x)$, replace $x$ with $2x$ in $t(x) = 5\sqrt{6x^2}$.

$t(x) = 5\sqrt{6x^2}$

t$(2x) = 5\sqrt{6(2x)^2}$

$= 5\sqrt{24x^2}$

$= 10|x|\sqrt{6}$

To find $t(7 + n)$, replace $x$ with $7 + n$ in $t(x) = 5\sqrt{6x^2}$.

$t(x) = 5\sqrt{6x^2}$

t$(7 + n) = 5\sqrt{6(7 + n)^2}$

$= 5|7 + n|\sqrt{6}$

**ANSWER:**

a. $20\sqrt{6}$

b. $10|x|\sqrt{6}$

c. $5|7 + n|\sqrt{6}$
1-1 Functions

38. DIGITAL AUDIO PLAYERS The sales of digital audio players, in millions of dollars, for a five-year period can be modeled using \( f(t) = 24t^2 - 93t + 78 \), where \( t \) is the year. The actual sales data are shown in the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 million</td>
</tr>
<tr>
<td>2</td>
<td>3 million</td>
</tr>
<tr>
<td>3</td>
<td>14 million</td>
</tr>
<tr>
<td>4</td>
<td>74 million</td>
</tr>
<tr>
<td>5</td>
<td>219 million</td>
</tr>
</tbody>
</table>

a. Find \( f(1) \) and \( f(5) \).
b. Do you think that the model is more accurate for the earlier years or the later years? Explain your reasoning.

SOLUTION:

a. To find \( f(1) \), replace \( t \) with 1 in \( f(t) = 24t^2 - 93t + 78 \).

\[ f(1) = 24(1)^2 - 93(1) + 78 \]
\[ = 24 - 93 + 78 \]
\[ = 9 \]

Therefore, according to the model, the sales for the first year were 9 million.

To find \( f(5) \), replace \( t \) with 5 in \( f(t) = 24t^2 - 93t + 78 \).

\[ f(5) = 24(5)^2 - 93(5) + 78 \]
\[ = 600 - 465 + 78 \]
\[ = 213 \]

Therefore, according to the model, the sales for the fifth year were 213 million.

b. Sample answer: I think the model is closer for the later years, which have the higher sales numbers, because 213 is within 2% of 219 and 9 is 800% larger than 1.

ANSWER:

a. $9 million; $213 million
b. Sample answer: I think the model is closer for the later years, which have the higher sales numbers, because 213 is within 2% of 219 and 9 is 800% larger than 1.

39. \( f(x) = \frac{8x + 12}{x^2 + 5x + 4} \)

State the domain of each function.

SOLUTION:

When the denominator of \( f(x) = \frac{8x + 12}{x^2 + 5x + 4} \) is zero, the expression is undefined.

\[ x^2 + 5x + 4 = 0 \]
\[ (x + 4)(x + 1) = 0 \]
\[ x = -4 \quad x = -1 \]

Therefore, the domain of this function is all real numbers except \( x = -4 \) and \( x = -1 \), or \((-\infty, -4) \cup (-4, -1) \cup (-1, \infty)\).

ANSWER:

\((-\infty, -4) \cup (-4, -1) \cup (-1, \infty)\)

40. \( g(x) = \frac{x + 1}{x^2 - 3x - 40} \)

SOLUTION:

When the denominator of \( g(x) = \frac{x + 1}{x^2 - 3x - 40} \) is zero, the expression is undefined.

\[ x^2 - 3x - 40 = 0 \]
\[ (x - 8)(x + 5) = 0 \]
\[ x = 8 \quad x = -5 \]

Therefore, the domain of \( g(x) \) is all real numbers except \( x = 8 \) and \( x = -5 \), or \((-\infty, -5) \cup (-5, 8) \cup (8, \infty)\).

ANSWER:

\((-\infty, -5) \cup (-5, 8) \cup (8, \infty)\)

41. \( g(a) = \sqrt{1 + a^2} \)

SOLUTION:

There is no value of \( a \) that will make the expression \( \sqrt{1 + a^2} \) undefined. Therefore, the domain of \( g(a) \) includes all real numbers or \((-\infty, \infty)\).

ANSWER:

\((-\infty, \infty)\)
1-1 Functions

42. \( h(x) = \sqrt{6 - x^2} \)

**SOLUTION:**
The square root of a negative number cannot be a real number, so \( 6 - x^2 \geq 0 \). If \( x \) is greater than \( \sqrt{6} \) or less than \( -\sqrt{6} \), the expression \( 6 - x^2 \) will be negative, and thus will not be a real number. Therefore, the domain of \( h(x) \) is \([ -\sqrt{6}, \sqrt{6} ]\).

**ANSWER:**
\([ -\sqrt{6}, \sqrt{6} ]\)

43. \( f(a) = \frac{5a}{4a - 1} \)

**SOLUTION:**
This function is defined only when \( 4a - 1 > 0 \) or \( a > 0.25 \). Therefore, the domain of \( f(a) \) is \((0.25, \infty)\).

**ANSWER:**
\((0.25, \infty)\)

44. \( g(x) = \frac{3}{\sqrt{x^2 - 16}} \)

**SOLUTION:**
This function is defined only when \( x^2 - 16 > 0 \). If \( x \) is greater than \( -4 \) or less than \( 4 \), the radicand will be negative, and thus will not be a real number. Therefore, the domain of \( g(x) \) is \(( -\infty, -4 ) \cup (4, \infty)\).

**ANSWER:**
\(( -\infty, -4 ) \cup (4, \infty)\)

45. \( f(x) = \frac{2}{x} + \frac{4}{x + 1} \)

**SOLUTION:**
This function is defined only when \( x > 0 \) and \( x + 1 > 0 \). Therefore, the function is defined for all real numbers except \( x = 0 \) and \( x = -1 \). So, the domain of \( f(x) \) is \(( -\infty, -1 ) \cup (-1, 0) \cup (0, \infty)\).

**ANSWER:**
\(( -\infty, -1 ) \cup (-1, 0) \cup (0, \infty)\)

46. \( g(x) = \frac{6}{x + 3} + \frac{2}{x - 4} \)

**SOLUTION:**
This function is defined only when \( x + 3 > 0 \) and \( x - 4 > 0 \). Therefore, the function is defined for all real numbers except \( x = -3 \) and \( x = 4 \). So, the domain of \( g(x) \) is \(( -\infty, -3 ) \cup (-3, 4) \cup (4, \infty)\).

**ANSWER:**
\(( -\infty, -3 ) \cup (-3, 4) \cup (4, \infty)\)

47. **PHYSICS** The period \( T \) of a pendulum is the time for one cycle and can be calculated using the formula \( T = 2\pi \sqrt{\frac{l}{g}} \), where \( l \) is the length of the pendulum and 9.8 is the gravitational acceleration due to gravity in meters per second squared. Is this formula a function of \( l \)? If so, determine the domain. If not, explain why not.

**SOLUTION:**
Yes; sample answer: The formula \( T = 2\pi \sqrt{\frac{l}{9.8}} \) is a function of \( l \) because the length of the pendulum must be positive. With this restriction, every value of \( l \) is now assigned to exactly one value of \( T \), and the domain of the function is \((0, \infty)\).

**ANSWER:**
Yes; sample answer: Because length must be positive, the domain of the function is \((0, \infty)\).
1-1 Functions

Find \( f(-5) \) and \( f(12) \) for each piecewise function.

48. \( f(x) = \begin{cases} 
-4x + 3 & \text{if } x < 3 \\
-x^2 & \text{if } 3 \leq x \leq 8 \\
3x^2 + 1 & \text{if } x > 8 
\end{cases} \)

\[ f(-5) = -4(-5) + 3 = 23 \]

\[ f(12) = 3(12)^2 + 1 = 433 \]

\[ \text{Answer: } 23; 433 \]

49. \( f(x) = \begin{cases} 
-5x^2 & \text{if } x < -6 \\
x^2 + x + 1 & \text{if } -6 \leq x \leq 12 \\
0.5x^3 - 4 & \text{if } x > 12 
\end{cases} \)

\[ f(-5) = -5(-5)^2 = -125 \]

\[ f(12) = (12)^2 + 12 + 1 = 157 \]

\[ \text{Answer: } 21; 157 \]

SOLUTION:

To find \( f(-5) \), use \( f(x) = -4x + 3 \).

\[ f(-5) = -4(-5) + 3 = 23 \]

To find \( f(12) \), use \( f(x) = 3x^2 + 1 \).

\[ f(12) = 3(12)^2 + 1 = 433 \]

\[ \text{Answer: } 23; 433 \]

50. \( f(x) = \begin{cases} 
2x^2 + 6x + 4 & \text{if } x < -4 \\
6 - x^2 & \text{if } -4 \leq x < 12 \\
14 & \text{if } x \geq 12 
\end{cases} \)

\[ f(-5) = 2(-5)^2 + 6(-5) + 4 = 24 \]

When \( x \geq 12 \), \( f(x) = 14 \), so \( f(12) = 14 \).

\[ \text{Answer: } 24; 14 \]

51. \( f(x) = \begin{cases} 
-15 & \text{if } x < -5 \\
\sqrt{x + 6} & \text{if } -5 \leq x \leq 10 \\
\frac{2}{x} + 8 & \text{if } x > 10 
\end{cases} \)

\[ f(-5) = \sqrt{(-5) + 6} = 1 \]

To find \( f(12) \), use \( f(x) = \frac{2}{x} + 8 \).

\[ f(12) = \frac{2}{12} + 8 = \frac{1}{6} + 8 = \frac{49}{6} \]

\[ \text{Answer: } 1; \frac{49}{6} \]
1-1 Functions

52. INCOME TAX Federal income tax for a person filing single in the United States in a recent year can be modeled using the following function, where \( x \) represents income and \( T(x) \) represents total tax.

\[
T(x) = \begin{cases} 
0.10x & \text{if } 0 \leq x \leq 7285 \\
782.5 + 0.15x & \text{if } 7285 < x \leq 31,850 \\
4386.25 + 0.25x & \text{if } 31,850 < x \leq 77,100 
\end{cases}
\]

a. Find \( T(7000) \), \( T(10,000) \), and \( T(50,000) \).
b. If a person’s annual income were $7285, what would his or her income tax be?

**SOLUTION:**

a. To find \( T(7000) \), use \( T(x) = 0.10x \).

\[
T(x) = 0.10(7000) = 700
\]

To find \( T(10,000) \), use \( T(x) = 782.5 + 0.15x \).

\[
T(x) = 782.5 + 0.15(10,000) = 782.5 + 1500 = 2282.5
\]

To find \( T(50,000) \), use \( T(x) = 4386.25 + 0.25x \).

\[
T(x) = 4386.25 + 0.25(50,000) = 4386.25 + 12500 = 16886.25
\]

b. To find \( T(7285) \), use \( T(x) = 0.10x \).

\[
T(x) = 0.10(7285) = 728.5
\]

Therefore, a person with an annual income of $7285 will have an income tax of $728.50.

**ANSWER:**

a. $700, $2282.5, $16,886.25
b. $728.50

53. PUBLIC TRANSPORTATION The nationwide use of public transportation can be modeled using the following function. The year 1996 is represented by \( t = 0 \), and \( P(t) \) represents passenger trips in millions.

\[
P(t) = \begin{cases} 
0.35t + 7.6 & \text{if } 0 \leq t \leq 5 \\
0.04t^2 - 0.6t + 11.6 & \text{if } 5 < t \leq 10 
\end{cases}
\]

a. Approximately how many passenger trips were there in 1999? in 2004?
b. State the domain of the function.

**SOLUTION:**

a. The year 1996 is represented by \( t = 0 \), so 1999 is represented by \( t = 3 \) and 2004 is represented by \( t = 8 \).

Because 3 is between 0 and 5, use \( P(t) = 0.35t + 7.6 \) to find \( P(3) \).

\[
P(3) = 0.35(3) + 7.6 = 1.05 + 7.6 = 8.65
\]

Because 8 is between 6 and 10, use \( P(t) = 0.04t^2 - 0.6t + 11.6 \) to find \( P(8) \).

\[
P(8) = 0.04(8)^2 - 0.6(8) + 11.6 = 2.56 - 4.8 + 11.6 = 9.36
\]

b. The intervals of the domain are \( 0 \leq t \leq 5 \) and \( 5 < t \leq 10 \). Therefore, the domain of \( P(t) \) includes integral values inside the interval [0, 10].

**ANSWER:**

a. 8.65 million; 9.36 million
b. integral values inside the interval [0, 10]
1-1 Functions

Use the vertical line test to determine whether each graph represents a function. Write yes or no. Explain your reasoning.

SOLUTION:

The graph passes the vertical line test. Therefore, the graph represents $y$ as a function of $x$.

ANSWER:
Yes; a vertical line would not pass through the graph more than once.

SOLUTION:

A vertical line at $x = 2$ intersects the graph at infinitely many points. Therefore, the graph does not represent $y$ as a function of $x$.

ANSWER:
No; a vertical line would pass through infinitely many points.

SOLUTION:

The graph passes the vertical line test. Therefore, the graph represents $y$ as a function of $x$.

ANSWER:
Yes; a vertical line would not pass through the graph more than once.
1-1 Functions

57. SOLUTION:

The $y$-axis is a vertical line that passes through two points of the graph, $(0, 0)$ and $(0, -4)$. Therefore, the graph does not represent $y$ as a function of $x$.

ANSWER:
No; the $y$-axis is a vertical line that passes through two points on the graph, $(0, 0)$ and $(0, -4)$.

58. TRIATHLON In a triathlon, athletes swim 2.4 miles, then bike 112 miles, and finally run 26.2 miles. Jesse’s average rates for each leg of a triathlon are shown in the table.

<table>
<thead>
<tr>
<th>Leg</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>swim</td>
<td>4 mph</td>
</tr>
<tr>
<td>bike</td>
<td>20 mph</td>
</tr>
<tr>
<td>run</td>
<td>6 mph</td>
</tr>
</tbody>
</table>

a. Write a piecewise function to describe the distance $D$ that Jesse has traveled in terms of time $t$. Round $t$ to the nearest tenth, if necessary.

b. State the domain of the function.

SOLUTION:

a. Use the distance equation $d = rt$ to find the time interval that corresponds to each leg of the race.

Swimming:
\[
d = rt
\]
\[
2.4 = 4t
\]
\[
2.4 \div 4 = t
\]
\[
0.6 = t
\]

So, the time interval that corresponds to the swimming portion of the race is $0 \leq t \leq 0.6$.

Biking:
\[
d = rt
\]
\[
112 = 20t
\]
\[
112 \div 20 = t
\]
\[
5.6 = t
\]

The time interval that corresponds to the biking portion of the race is $0.6 < t \leq 0.6 + 5.6$ or $0.6 < t \leq 6.2$.

Running:
\[
d = rt
\]
\[
26.2 = 6t
\]
\[
26.2 \div 6 = t
\]
\[
4.4 = t
\]

The time interval that corresponds to the running portion of the race is $6.2 < t \leq 6.2 + 4.4$ or $6.2 < t \leq 10.6$.

The distance traveled during the first leg of the race is given by $D(t) = 4t$. The distance traveled during the second leg of the race is given by the sum of the distance traveled during the first leg of the race $d_1$ (2.4 miles) and the distance that is being traveled during the second leg of the race $d_2$, which is given by $d_2 = 20(t - 0.6)$.

\[
D(t) = d_1 + d_2
\]
\[
= 2.4 + 20(t - 0.6)
\]
\[
= 2.4 + 20t - 12
\]
\[
= 20t - 9.6
\]

The distance traveled during the third leg of the race is given by the sum of the distance traveled during the first leg of the race, the distance traveled during the second leg of the race, and the distance that is being traveled during the third leg of the race. The distance traveled in the third leg of the race is given by $d_3 = 6(t - 6.2)$.

\[
D(t) = d_1 + d_2 + d_3
\]
\[
= 2.4 + 112 + 6(t - 6.2)
\]
\[
= 114.4 + 6t - 37.2
\]
\[
= 6t + 77.2
\]

A piecewise function that describes the distance $D$ that Jesse traveled in terms of time $t$ is shown below.

\[
D(t) = \begin{cases} 
4t & \text{if } 0 \leq t \leq 0.6 \\
20t - 9.6 & \text{if } 0.6 < t \leq 6.2 \\
6t + 77.2 & \text{if } 6.2 < t \leq 10.6 
\end{cases}
\]

b. The domain of the function represents the interval of time beginning when Jesse started the
1-1 Functions

first leg of the marathon and ending when she completed the marathon. Jessie started the marathon at \( t = 0 \) and finished the marathon at \( t = 10.6 \). Therefore, the domain is \([0, 10.6]\).

**ANSWER:**

a. 

\[
D(t) = \begin{cases} 
4t & \text{if } 0 \leq t \leq 0.6 \\
20t - 9.6 & \text{if } 0.6 < t \leq 6.2 \\
6t + 77.2 & \text{if } 6.2 < t \leq 10.6 
\end{cases} 
\]

b. \([0, 10.6]\)

59. **ELECTIONS** Describe the set of presidential election years beginning in 1792 in interval notation or in set-builder notation. Explain your reasoning.

**SOLUTION:**

Sample answer: Because presidential elections are held every 4 years, and do not have a finite end, it is impractical to display the set in interval notation. If set-builder notation is used, the interval can be taken into account and a finite interval is not necessary. Therefore, the set of presidential election years beginning in 1792 can be described in set-builder notation as \( \{x \mid x = 4n + 1792, n \in \mathbb{W}\} \).

**ANSWER:**

\( \{x \mid x = 4n + 1792, n \in \mathbb{W}\} \); Sample answer: Because presidential elections are held every 4 years, and do not have a finite end, it is impractical to display the set in interval notation. If set-builder notation is used, the interval can be taken into account and a finite interval is not necessary.

60. **CONCESSIONS** The number of students working the concession stands at a football game can be represented by \( f(x) = \frac{x}{50} \), where \( x \) is the number of tickets sold. Describe the relevant domain of the function.

**SOLUTION:**

The domain of the function \( f(x) = \frac{x}{50} \) is all real numbers or \((-\infty, \infty)\). Because the number of tickets sold must be a whole number, the relevant domain of \( f(x) \) is all whole numbers from 0 to the capacity of the stadium.

**ANSWER:**

The domain of the function is the set of whole numbers from 0 to the capacity of the stadium.

61. **ATTENDANCE** The Chicago Cubs franchise has been in existence since 1874. The total season attendance for its home games can be modeled by \( f(x) = 70,050x - 137,400,000 \), where \( x \) represents the year. Describe the relevant domain of the function.

**SOLUTION:**

Because the number of years must be a whole number, the relative domain of the function is the set of whole numbers greater than or equal to 1874, or \( D = \{x \mid x \geq 1874, x \in \mathbb{W}\} \).

**ANSWER:**

\( D = \{x \mid x \geq 1874, x \in \mathbb{W}\} \)
1-1 Functions

62. **ACCOUNTING** A business’ assets, such as equipment, wear out or depreciate over time. One way to calculate depreciation is the straight-line method, using the value of the estimated life of the asset. Suppose \( v(t) = 10,440 - 290t \) describes the value \( v(t) \) of a copy machine after \( t \) months. Describe the relevant domain of the function.

**SOLUTION:**
Because the number of months must be a nonnegative number, \( t \geq 0 \).

Graph the function. From the graph of \( v(t) \) shown below, the relevant domain is all real numbers greater than or equal to zero and less than or equal to 36. Therefore, \( D = \{ t \mid 0 \leq t \leq 36, t \in \mathbb{R} \} \).

![](image)

**ANSWER:**
\( D = \{ t \mid 0 \leq t \leq 36, t \in \mathbb{R} \} \)

Find \( f(a) \), \( f(a + h) \), and \( \frac{f(a + h) - f(a)}{h} \) if \( h \neq 0 \).

63. \( f(x) = -5 \)

**SOLUTION:**
Since there is no independent variable \( x \), \( f(a) = -5 \) and \( f(a + h) = -5 \).

Use the values that you found for \( f(a) \) and \( f(a + h) \) to find \( \frac{f(a + h) - f(a)}{h} \).

\[
\frac{f(a + h) - f(a)}{h} = \frac{-5 - (-5)}{h} = \frac{-5 + 5}{h} = \frac{0}{h} \text{ or } 0
\]

**ANSWER:**
\(-5; -5; 0\)

64. \( f(x) = \sqrt{x} \)

**SOLUTION:**
To find \( f(a) \), replace \( x \) with \( a \) in \( f(x) = \sqrt{x} \).

\[
f(x) = \sqrt{x} \quad f(a) = \sqrt{a}
\]

To find \( f(a + h) \), replace \( x \) with the expression \( a + h \) in \( f(x) = \sqrt{x} \).

\[
f(x) = \sqrt{x} \quad f(a + h) = \sqrt{a + h}
\]

Use the expressions that you found for \( f(a) \) and \( f(a + h) \) to find \( \frac{f(a + h) - f(a)}{h} \).

\[
\frac{f(a + h) - f(a)}{h} = \frac{\sqrt{a + h} - \sqrt{a}}{h}
\]

**ANSWER:**
\( \sqrt{a}; \sqrt{a + h}; \frac{\sqrt{a + h} - \sqrt{a}}{h} \)
1-1 Functions

65. \( f(x) = \frac{1}{x+4} \)

**SOLUTION:**

To find \( f(a) \), replace \( x \) with \( a \) in \( f(x) = \frac{1}{x+4} \).

\[
\begin{align*}
  f(x) &= \frac{1}{x+4} \\
  f(a) &= \frac{1}{a+4}
\end{align*}
\]

To find \( f(a+h) \), replace \( x \) with the expression \( a+h \) in \( f(x) = \sqrt{x} \).

\[
\begin{align*}
  f(x) &= \frac{1}{x+4} \\
  f(a+h) &= \frac{1}{a+h+4}
\end{align*}
\]

Use the expressions that you found for \( f(a) \) and \( f(a+h) \) to find \( \frac{f(a+h) - f(a)}{h} \).

\[
\begin{align*}
  f(a+h) - f(a) &= \frac{1}{a+h+4} - \frac{1}{a+4} \\
  &= \frac{a+4 - (a+h+4)}{(a+4)(a+h+4)} \\
  &= \frac{a+4 - (a+h+4)}{(a+4)(a+h+4)} \\
  &= \frac{-h}{(a+4)(a+h+4)} \\
  &= \frac{-h}{a^2 + ah + 8a + 4h + 16} \\
  &= \frac{-1}{h}
\end{align*}
\]

**ANSWER:**

\[
\frac{1}{a+4}; \quad \frac{1}{a+h+4}; \quad \frac{-1}{a^2 + ah + 8a + 4h + 16}
\]

66. \( f(x) = \frac{2}{5-x} \)

**SOLUTION:**

To find \( f(a) \), replace \( x \) with \( a \) in \( f(x) = \frac{2}{5-x} \).

\[
\begin{align*}
  f(x) &= \frac{2}{5-x} \\
  f(a) &= \frac{2}{5-a}
\end{align*}
\]

To find \( f(a+h) \), replace \( x \) with the expression \( a+h \) in \( f(x) = \frac{2}{5-x} \).

\[
\begin{align*}
  f(x) &= \frac{2}{5-x} \\
  f(a+h) &= \frac{2}{5-(a+h)}
\end{align*}
\]

Use the expressions that you found for \( f(a) \) and \( f(a+h) \) to find \( \frac{f(a+h) - f(a)}{h} \).

\[
\begin{align*}
  f(a+h) - f(a) &= \frac{2}{5-(a+h)} - \frac{2}{5-a} \\
  &= \frac{2(5-a) - 2(5-(a+h))}{h} \\
  &= \frac{2(5-a) - 2(5-a+h)}{h} \\
  &= \frac{2(5-a) - (5-a-h)}{h} \\
  &= \frac{2(a-ah-5h+25)}{h} \\
  &= \frac{2(a-ah-5h+25)}{2h} \\
  &= \frac{a-ah-5h+25}{h}
\end{align*}
\]

**ANSWER:**

\[
\frac{2}{5-a}; \quad \frac{2}{5-a-h}; \quad \frac{2}{a^2 - 10a + ah - 5h + 25}
\]
1-1 Functions

67. \( f(x) = x^2 - 6x + 8 \)

**SOLUTION:**

To find \( f(a) \), replace \( x \) with \( a \) in \( f(x) = x^2 - 6x + 8 \).

\[ f(x) = x^2 - 6x + 8 \]

\[ f(a) = a^2 - 6a + 8 \]

To find \( f(a + h) \), replace \( x \) with the expression \( a + h \) in \( f(x) = x^2 - 6x + 8 \).

\[ f(x) = x^2 - 6x + 8 \]

\[ f(a + h) = (a + h)^2 - 6(a + h) + 8 \]

\[ f(a + h) = a^2 + 2ah + h^2 - 6a - 6h + 8 \]

Use the expressions that you found for \( f(a) \) and \( f(a + h) \) to find \( \frac{f(a + h) - f(a)}{h} \).

\[ \frac{f(a + h) - f(a)}{h} = \frac{a^2 + 2ah + h^2 - 6a - 6h + 8 - (a^2 - 6a + 8)}{h} \]

\[ = \frac{2ah + h^2}{h} \]

\[ = \frac{h(2a + h)}{h} \]

\[ = 2a + h \]

**ANSWER:**

\( a^2 - 6a + 8; \quad a^2 + 2ah + h^2 - 6a - 6h + 8; \quad 2a + h \)

68. \( f(x) = -\frac{1}{4}x + 6 \)

**SOLUTION:**

To find \( f(a) \), replace \( x \) with \( a \) in \( f(x) = -\frac{1}{4}x + 6 \).

\[ f(x) = -\frac{1}{4}x + 6 \]

\[ f(a) = -\frac{1}{4}a + 6 \]

To find \( f(a + h) \), replace \( x \) with the expression \( a + h \) in \( f(x) = -\frac{1}{4}x + 6 \).

\[ f(x) = -\frac{1}{4}x + 6 \]

\[ f(a + h) = -\frac{1}{4}(a + h) + 6 \]

\[ f(a + h) = -\frac{a + h}{4} + 6 \]

Use the expressions that you found for \( f(a) \) and \( f(a + h) \) to find \( \frac{f(a + h) - f(a)}{h} \).

\[ \frac{f(a + h) - f(a)}{h} = \frac{-\frac{a + h}{4} + 6 - (-\frac{1}{4}a + 6)}{h} \]

\[ = \frac{-a - h}{4} + \frac{1}{4} \]

\[ = \frac{-a - h + a}{4} \]

\[ = \frac{-h}{4} \]

\[ = \frac{1}{4} \]

**ANSWER:**

\( -\frac{1}{4}a + 6; \quad -\frac{a - h}{4} + 6; \quad -\frac{1}{4} \)
1-1 Functions

69. \( f(x) = -x^5 \)

**SOLUTION:**

To find \( f(a) \), replace \( x \) with \( a \):

\[ f(x) = -x^5 \]
\[ f(a) = -a^5 \]

To find \( f(a + h) \), replace \( x \) with the expression \( a + h \):

\[ f(x) = -x^5 \]
\[ f(a + h) = -(a^5 + 5a^4h + 10a^3h^2 + 10a^2h^3 + 5ah^4 + h^5) \]
\[ f(a + h) = -a^5 - 5a^4h - 10a^3h^2 - 10a^2h^3 - 5ah^4 - h^5 \]

Use the expressions that you found for \( f(a) \) and \( f(a + h) \) to find

\[ \frac{f(a + h) - f(a)}{h} \]

\[ = \frac{-a^5 - 5a^4h - 10a^3h^2 - 10a^2h^3 - 5ah^4 - h^5 - (-a^5)}{h} \]
\[ = -5a^4h - 10a^3h^2 - 10a^2h^3 - 5ah^4 - h^5 \]

**ANSWER:**

\(-a^5; -a^5 - 5a^4h - 10a^3h^2 - 10a^2h^3 - 5ah^4 - h^5; -5a^4 - 10a^3h - 10a^2h^2 - 5ah^3 - h^4\)

70. \( f(x) = x^3 + 9 \)

**SOLUTION:**

To find \( f(a) \), replace \( x \) with \( a \):

\[ f(x) = x^3 + 9 \]
\[ f(a) = a^3 + 9 \]

To find \( f(a + h) \), replace \( x \) with the expression \( a + h \):

\[ f(x) = x^3 + 9 \]
\[ f(a + h) = (a + h)^3 + 9 \]
\[ f(a + h) = a^3 + 3a^2h + 3ah^2 + h^3 + 9 \]

Use the expressions that you found for \( f(a) \) and \( f(a + h) \) to find

\[ \frac{f(a + h) - f(a)}{h} \]

\[ = \frac{a^3 + 3a^2h + 3ah^2 + h^3 + 9 - (a^3 + 9)}{h} \]
\[ = \frac{3a^2h + 3ah^2 + h^3}{h} \]
\[ = 3a^2h + 3ah + h^2 \]

**ANSWER:**

\( a^3 + 9; a^3 + 3a^2h + 3ah^2 + h^3 + 9; 3a^2 + 3ah + h^2 \)
1-1 Functions

71. \( f(x) = 7x - 3 \)

**SOLUTION:**
To find \( f(a) \), replace \( x \) with \( a \) in \( f(x) = 7x - 3 \).
\[
\begin{align*}
    f(x) &= 7x - 3 \\
    f(a) &= 7a - 3
\end{align*}
\]
To find \( f(a + h) \), replace \( x \) with the expression \( a + h \) in \( f(x) = 7x - 3 \).
\[
\begin{align*}
    f(x) &= 7x - 3 \\
    f(a + h) &= 7(a + h) - 3 \\
    f(a + h) &= 7a + 7h - 3
\end{align*}
\]
Use the expressions that you found for \( f(a) \) and \( f(a + h) \) to find \( \frac{f(a + h) - f(a)}{h} \).
\[
\begin{align*}
    f(a + h) - f(a) &= 7a + 7h - 3 - (7a - 3) \\
    &= 7a + 7h - 3 - 7a + 3 \\
    &= 7h \\
    \frac{f(a + h) - f(a)}{h} &= \frac{7h}{h} \\
    &= 7
\end{align*}
\]
**ANSWER:**
\( 7a - 3; 7a + 7h - 3; 7 \)

72. \( f(x) = 5x^2 \)

**SOLUTION:**
To find \( f(a) \), replace \( x \) with \( a \) in \( f(x) = 5x^2 \).
\[
\begin{align*}
    f(x) &= 5x^2 \\
    f(a) &= 5a^2
\end{align*}
\]
To find \( f(a + h) \), replace \( x \) with the expression \( a + h \) in \( f(x) = 5x^2 \).
\[
\begin{align*}
    f(x) &= 5x^2 \\
    f(a + h) &= 5(a + h)^2 \\
    f(a + h) &= 5(a^2 + 2ah + h^2) \\
    f(a + h) &= 5a^2 + 10ah + 5h^2
\end{align*}
\]
Use the expressions that you found for \( f(a) \) and \( f(a + h) \) to find \( \frac{f(a + h) - f(a)}{h} \).
\[
\begin{align*}
    f(a + h) - f(a) &= 5a^2 + 10ah + 5h^2 - 5a^2 \\
    &= 10ah + 5h^2 \\
    \frac{f(a + h) - f(a)}{h} &= \frac{10ah + 5h^2}{h} \\
    &= 10a + 5h \\
\end{align*}
\]
**ANSWER:**
\( 5a^2; 5a^2 + 10ah + 5h^2; 10a + 5h \)
1-1 Functions

73. \( f(x) = x^3 \)

**SOLUTION:**

To find \( f(a) \), replace \( x \) with \( a \) in \( f(x) = x^3 \).

\[
\begin{align*}
  f(x) &= x^3 \\
  f(a) &= a^3
\end{align*}
\]

To find \( f(a + h) \), replace \( x \) with the expression \( a + h \) in \( f(x) = x^3 \).

\[
\begin{align*}
  f(x) &= x^3 \\
  f(a + h) &= (a + h)^3 \\
  f(a + h) &= a^3 + 3a^2h + 3ah^2 + h^3
\end{align*}
\]

Use the expressions that you found for \( f(a) \) and \( f(a + h) \) to find \( \frac{f(a + h) - f(a)}{h} \).

\[
\begin{align*}
  \frac{f(a + h) - f(a)}{h} &= \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h} \\
  &= \frac{3a^2h + 3ah^2 + h^3}{h} \\
  &= \frac{h(3a^2 + 3ah + h^2)}{h} \\
  &= 3a^2 + 3ah + h^2
\end{align*}
\]

**ANSWER:**

\( a^3; a^3 + 3a^2h + 3ah^2 + h^3; 3a^2 + 3ah + h^2 \)

74. \( f(x) = 11 \)

**SOLUTION:**

Because there is no independent variable \( x \), \( f(a) = 11 \) and \( f(a + h) = 11 \).

Use the values that you found for \( f(a) \) and \( f(a + h) \) to find \( \frac{f(a + h) - f(a)}{h} \).

\[
\begin{align*}
  \frac{f(a + h) - f(a)}{h} &= \frac{11 - (11)}{h} \\
  &= \frac{11 - 11}{h} \text{ or } 0
\end{align*}
\]

**ANSWER:**

11; 11; 0

75. **MAIL** The U.S. Postal Service requires that envelopes have an aspect ratio (length divided by height) of 1.3 to 2.5, inclusive. The minimum allowable length is 5 inches and the maximum allowable length is \( 11 \frac{1}{2} \) inches.

**a.** Write the area of the envelope \( A \) as a function of length \( l \) if the aspect ratio is 1.8. State the domain of the function.

**b.** Write the area of the envelope \( A \) as a function of height \( h \) if the aspect ratio is 2.1. State the domain of the function.

**c.** Find the area of an envelope with the maximum height at the maximum aspect ratio.

**SOLUTION:**

**a.** The aspect ratio of the envelope is equal to 1.8, so \( \frac{l}{h} = 1.8 \), and \( h = \frac{l}{1.8} \). The area of the envelope \( A = lh \). Substitute \( h = \frac{l}{1.8} \) into the area equation for \( h \).

\[
A = lh
= \left( \frac{l}{1.8} \right) l
= \frac{l^2}{1.8}
\]

So, \( A(l) = \frac{l^2}{1.8} \).

The domain of the function includes all real numbers that are greater than or equal to the minimum allowable length of 5 inches and less than or equal to the maximum allowable length of 11.5 inches. So, in interval notation, the domain is \([5, 11.5]\).

**b.** If the aspect ratio of the envelope is equal to 2.1, \( \frac{l}{h} = 2.1 \), and \( l = 2.1h \). The area of the envelope \( A = lh \). Substitute \( l = 2.1h \) into the area equation for \( l \).

\[
A = lh
= (2.1h)h
= 2.1h^2
\]

So, \( A(h) = 2.1h^2 \).

Use the equation for the aspect ratio to find the
1-1 Functions

minimum and maximum allowable heights for the envelope.
minimum height:
\[
\frac{\ell}{h} = 2.1
\]
\[
\frac{5}{h} = 2.1
\]
\[
5 = 2.1h
\]
2.4 = h
maximum height:
\[
\frac{\ell}{h} = 2.1
\]
\[
\frac{11.5}{h} = 2.1
\]
11.5 = 2.1h
5.5 = h

The domain of the function includes all real numbers that are greater than or equal to the minimum allowable height of 2.4 inches and less than or equal to the maximum allowable height of 5.5 inches. So, in interval notation, the domain is [2.4, 5.5].

c. In the problem statement it is given that the maximum aspect ratio is 2.5. Because the aspect ratio is equal to the length divided by the height, the aspect ratio will be greatest when the length is maximized and the height is minimized. It is also given that the maximum allowable length is 11.5 inches.

Find the value of h when the aspect ratio is 2.5 and the length is 11.5
\[
\frac{\ell}{h} = 2.5
\]
\[
\frac{11.5}{h} = 2.5
\]
11.5 = 2.5h
4.6 = h

Substitute the values that you found for \( l \) and \( h \) into the area formula.
\[
A = \ell h
\]
\[
= (11.5)(4.6)
\]
\[
= 52.9
\]

Therefore, the area of an envelope with the maximum height at the maximum aspect ratio is 52.9 in\(^2\).

ANSWER:

a. \( A(\ell) = \frac{\ell^2}{1.8} \); [5, 11.5]

b. \( A(h) = 2.1h^2 \); [2.4, 5.5]

c. 52.9 in\(^2\)

76. GEOMETRY Consider the circle below with area \( A \) and circumference \( C \).

a. Represent the area of the circle as a function of its circumference.
b. Find \( A(0.5) \) and \( A(4) \).
c. What do you notice about the area as the circumference increases?

SOLUTION:

a. The area of a circle is given by \( A = \pi r^2 \) and the circumference is given by \( C = 2\pi r \).

First, solve the circumference equation for \( r \).
\[
C = 2\pi r
\]
\[
\frac{C}{2\pi} = r
\]

Next, replace \( r \) with \( \frac{C}{2\delta} \) in the equation for the area of a circle.
\[
A = \pi r^2
\]
\[
= \pi \left( \frac{C}{2\pi} \right)^2
\]
\[
= \pi \left( \frac{C^2}{4\pi^2} \right)
\]
\[
= \frac{C^2}{4\pi}
\]

Therefore, the area of a circle as a function of its circumference can be written as \( A = \frac{C^2}{4\delta} \).

b. To find \( A(0.5) \), replace \( C \) with 0.5 in the expression \( A = \frac{C^2}{4\delta} \).

\[
A = \frac{C^2}{4\pi}
\]
\[
= \frac{(0.5)^2}{4\pi}
\]
\[
= \frac{0.25}{4\pi}
\]
\[
= 0.02
\]

To find \( A(4) \), replace \( C \) with 4 in the expression \( A = \frac{C^2}{4\delta} \).
1-1 Functions

\[ \frac{C^2}{4\delta} \]

\[ A = \frac{C^2}{4\pi} \]
\[ = \frac{(4)^2}{4\pi} \]
\[ = \frac{16}{4\pi} \]
\[ \approx 1.27 \]

c. As the circumference increases, the value of the expression \( \frac{C^2}{4\delta} \) increases, therefore the area will also increase.

**ANSWER:**

a. \( A = \frac{C^2}{4\delta} \)

b. 0.02, 1.27

c. As the circumference increases, the area also increases.

Determine whether each equation is a function of \( x \). Explain.

77. \( x = |y| \)

**SOLUTION:**

Sample answer: Each range value is paired with two possible domain values because it is necessary to take both the positive and negative values of the absolute value of \( x \) when solving the equation for \( y \). Therefore, the equation does not represent a function.

**ANSWER:**
No; sample answer: Most nonnegative \( x \)-values are paired with two \( y \)-values because it is necessary to take both the positive and negative values of the absolute value of \( x \) when solving the equation for \( y \).

78. \( x = y^3 \)

**SOLUTION:**

Sample answer: Each range value is paired with exactly one domain value, so the equation represents a function.

**ANSWER:**
Yes; sample answer: Each \( x \)-value is paired with exactly one \( y \)-value, so the equation represents a function.

79. MULTIPLE REPRESENTATIONS In this problem, you will investigate the range of a function.

a. GRAPHICAL Use a graphing calculator to graph \( f(x) = x^n \) for whole-number values of \( n \) from 1 to 6, inclusive.

b. TABULAR Predict the range of each function based on the graph, and tabulate each value of \( n \) and the corresponding range.

c. VERBAL Make a conjecture about the range of \( f(x) \) when \( n \) is even.

d. VERBAL Make a conjecture about the range of \( f(x) \) when \( n \) is odd.

**SOLUTION:**

a.

b.
1-1 Functions

<table>
<thead>
<tr>
<th>( n )</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((\infty, \infty))</td>
</tr>
<tr>
<td>2</td>
<td>([0, \infty))</td>
</tr>
<tr>
<td>3</td>
<td>((-\infty, \infty))</td>
</tr>
<tr>
<td>4</td>
<td>([0, \infty))</td>
</tr>
<tr>
<td>5</td>
<td>((-\infty, \infty))</td>
</tr>
<tr>
<td>6</td>
<td>([0, \infty))</td>
</tr>
</tbody>
</table>

c. Sample answer: When \( n = 2, n = 4, \) and \( n = 6 \) the range is \([0, \infty)\). Therefore, when \( n \) is even in \( f(x) = x^n \), the range is \([0, \infty)\).

d. Sample answer: When \( n = 1, n = 3, \) and \( n = 5 \) the range is \((-\infty, \infty)\). Therefore, when \( n \) is odd in \( f(x) = x^n \), the range is \((-\infty, \infty)\).

**ANSWER:**

80. ERROR ANALYSIS Ana and Mason are evaluating \( f(x) = \frac{2}{x^2 - 4} \). Ana thinks that the domain of the function is \((-\infty, -2)\) or \((1, 1)\) or \((2, \infty)\). Mason thinks that the domain is \( \{x \mid x \neq -2, x \neq 2, x \in \mathbb{R}\} \). Is either of them correct? Explain.

**SOLUTION:**

Sample answer: When the denominator of \( \frac{2}{x^2 - 4} \) is zero, the expression is undefined. Solve \( x^2 - 4 = 0 \).

\[
x^2 - 4 = 0
\]
\[
x^2 = 4
\]
\[
x = \pm 2
\]
So, the domain of \( f(x) \) is all real numbers except \( x = -2 \) and \( x = 2 \). In set-builder notation the domain can be written as \( \{x \mid x \neq -2, x \neq 2, x \in \mathbb{R}\} \), and in interval notation the domain can be written as \((-\infty, -2) \cup (-2, 2) \cup (2, \infty)\). Therefore, Mason’s answer is correct.

**ANSWER:**

Mason; sample answer: The domain is \((-\infty, -2) \cup (-2, 2) \cup (2, \infty)\) or \( \{x \mid x \neq -2, x \neq 2, x \in \mathbb{R}\} \).
81. **Writing in Math** Write the domain of \( f(x) = \frac{1}{(x+3)(x+1)(x-5)} \) in interval notation and in set-builder notation. Which notation do you prefer? Explain.

**SOLUTION:**
Sample answer: When the denominator of \( \frac{1}{(x+3)(x+1)(x-5)} \) is zero, the expression is undefined. Therefore, \( f(x) \) is undefined for \( x = -3 \), \( x = -1 \), and \( x = 5 \). In interval notation, the domain can be described as \((\infty, -3) \cup (-3, -1) \cup (-1, 5) \cup (5, \infty)\), and in set-builder notation the domain can be described as \( \{x \mid x \neq -3, x \neq -1, x \neq 5, x \in \mathbb{R} \} \). In this case, set-builder notation is preferential because it is more concise to list the three real numbers to which \( x \) cannot be equal rather than the four intervals of which \( x \) is an element.

**ANSWER:**
\((\infty, -3) \cup (-3, -1) \cup (-1, 5) \cup (5, \infty); \{x \mid x \neq -3, x \neq -1, x \neq 5, x \in \mathbb{R} \}\); Sample answer: In this case, set-builder notation is preferential because it is more concise to list the three real numbers to which \( x \) cannot be equal rather than the four intervals of which \( x \) is an element.

82. **CHALLENGE** \( G(x) \) is a function for which \( G(1) = 1 \), \( G(2) = 2 \), \( G(3) = 3 \), and \( G(x + 1) = \frac{G(x)G(x + 1) + 1}{G(x)} \) for \( x \geq 3 \). Find \( G(6) \).

**SOLUTION:**
First, find \( G(4) \) by replacing \( x \) with 3 in \( G(x + 1) = \frac{G(x)G(x + 1) + 1}{G(x)} \).

\[
\begin{align*}
G(x + 1) &= \frac{G(x)G(x + 1) + 1}{G(x)} \\
G(3 + 1) &= \frac{G(3)G(3 + 1) + 1}{G(3)} \\
G(4) &= \frac{G(1)G(2) + 1}{G(3)} \\
G(4) &= \frac{1 \cdot 2 + 1}{3} = \frac{3}{3} \text{ or 1} \\
\end{align*}
\]

Find \( G(5) \).

\[
\begin{align*}
G(x + 1) &= \frac{G(x)G(x + 1) + 1}{G(x)} \\
G(4 + 1) &= \frac{G(4)G(4 + 1) + 1}{G(4)} \\
G(5) &= \frac{G(2)G(3) + 1}{G(4)} \\
G(5) &= \frac{2 \cdot 3 + 1}{1} = 7 \text{ or 7} \\
\end{align*}
\]

Find \( G(6) \).

\[
\begin{align*}
G(x + 1) &= \frac{G(x)G(x + 1) + 1}{G(x)} \\
G(5 + 1) &= \frac{G(5)G(5 + 1) + 1}{G(5)} \\
G(6) &= \frac{G(3)G(4) + 1}{G(5)} \\
G(6) &= \frac{3 + 1}{7} = \frac{4}{7} \\
G(6) &= \frac{4}{7} \\
\end{align*}
\]

**ANSWER:**
\[
\frac{4}{7}
\]
1-1 Functions

REASONING Determine whether each statement is true or false given a function from set $X$ to set $Y$. If a statement is false, rewrite it to make a true statement.

83. Every element in $X$ must be matched with only one element in $Y$.

SOLUTION:
For a relation to be a function, every element in $X$ must be matched with only one element in $Y$. Therefore, the statement is true.

Answer:
true

84. Every element in $Y$ must be matched with an element in $X$.

SOLUTION:
False; sample answer: Every element in $X$ must be matched with exactly one element in $Y$.

Answer:
False; sample answer: Every element in $X$ must be matched with exactly one element in $Y$.

85. Two or more elements in $X$ may not be matched with the same element in $Y$.

SOLUTION:
False; sample answer: Two or more elements in $X$ may be matched with the same element in $Y$.

Answer:
False; sample answer: Two or more elements in $X$ may be matched with the same element in $Y$.

86. Two or more elements in $Y$ may not be matched with the same element in $X$.

SOLUTION:
For a relation to be a function, every element in $X$ must be matched with only one element in $Y$. Therefore, the statement is true.

Answer:
true

Writing in Math Explain how you can identify a function described as each of the following.

87. a verbal description of inputs and outputs

SOLUTION:
Sample answer: If each of the possible inputs will be assigned to exactly one output in the verbal description, the relation is a function.

Answer:
Sample answer: If each of the possible inputs is assigned to exactly one output in the verbal description, the relation is a function.

88. a set of ordered pairs

SOLUTION:
Sample answer: If each $x$-coordinate in the list of ordered pairs is paired with a distinct $y$-coordinate, the relation is a function.

Answer:
Sample answer: If each $x$-coordinate in the set of ordered pairs is paired with a unique $y$-coordinate, the relation is a function.

89. a table of values

SOLUTION:
Sample answer: If each input value in the table is paired with a distinct output value, the relation is a function.

Answer:
Sample answer: If each input value in the table is paired with a unique output value, the relation is a function.

90. a graph

SOLUTION:
Sample answer: If a vertical line drawn at any $x$-value on the graph intersects the graph exactly once, the relation is a function.

Answer:
Sample answer: If a vertical line drawn at any $x$-value on the graph intersects the graph exactly once, the relation is a function.
1-1 Functions

91. an equation

SOLUTION:
Sample answer: If each \( x \)-value will be paired with exactly one \( y \)-value after the equation is solved for \( y \), then the relation is a function.

ANSWER:
Sample answer: If each \( x \)-value can be paired with exactly one \( y \)-value after the equation is solved for \( y \), the relation is a function.

Find the standard deviation of each population of data.

92. \{200, 476, 721, 579, 152, 158\}

SOLUTION:
First, find the mean of the data.
\[
\mu = \frac{\sum X}{n} = \frac{200 + 476 + \ldots + 158}{6} = 381
\]

Use the mean to find the standard deviation.
\[
\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{n}} \\
= \sqrt{\frac{(200 - 381)^2 + (476 - 381)^2 + \ldots + (158 - 381)^2}{6}} \approx 223.14
\]

ANSWER:
223.14

93. \{5.7, 5.7, 5.6, 5.5, 5.3, 4.9, 4.4, 4.0, 4.0, 3.8\}

SOLUTION:
First, find the mean of the data.
\[
\mu = \frac{\sum X}{n} = \frac{5.7 + 5.7 + \ldots + 3.8}{10} = 4.89
\]

Use the mean to find the standard deviation.
\[
\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{n}} \\
= \sqrt{\frac{(5.7 - 4.89)^2 + (5.7 - 4.89)^2 + \ldots + (5.7 - 4.89)^2}{10}} \approx 0.73
\]

ANSWER:
0.73

94. \{369, 398, 381, 392, 406, 413, 376, 454, 420, 385, 402, 446\}

SOLUTION:
First, find the mean of the data.
\[
\mu = \frac{\sum X}{n} = \frac{369 + 398 + \ldots + 446}{12} = 403.5
\]

Use the mean to find the standard deviation.
\[
\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{n}} \\
= \sqrt{\frac{(369 - 403.5)^2 + (398 - 403.5)^2 + \ldots + (446 - 403.5)^2}{12}} \approx 25.31
\]

ANSWER:
25.31
1-1 Functions

95. **BASEBALL** How many different 9-player teams can be made if there are 3 players who can only play catcher, 4 players who can only play first base, 6 players who can only pitch, and 14 players who can play in any of the remaining 6 positions?

*SOLUTION:

In this problem, you need to find the number of combinations of $n$ players taken $r$ at a time for each position, as well as those that can play the remaining 6 positions, separately.

Find the combination of 3 players taken 1 at a time.

\[
\binom{n}{r} = \frac{n!}{(n-r)!r!}
\]

\[
\binom{3}{1} = \frac{3!}{(3-1)!1!} = 3
\]

\[
\binom{3}{1} = \frac{3!}{2!} = 3
\]

Find the combination of 4 players taken 1 at a time.

\[
\binom{4}{1} = \frac{4!}{(4-1)!1!} = 4
\]

\[
\binom{4}{1} = \frac{4!}{3!} = 4
\]

Find the combination of 6 players taken 1 at a time.

\[
\binom{6}{1} = \frac{6!}{(6-1)!1!} = 6
\]

\[
\binom{6}{1} = \frac{6!}{5!} = 6
\]

Next, find the combination of 14 players taken 6 at a time.

\[
\binom{14}{6} = \frac{14!}{(14-6)!6!}
\]

\[
\binom{14}{6} = \frac{14!}{8!6!} = 3003
\]

Use the Fundamental Counting Principle to find the number of 9-player teams that can be made.

\[
\text{number of teams} = n_1 \cdot n_2 \cdot n_3 \cdot n_4
\]

\[
\text{number of teams} = 3 \cdot 4 \cdot 6 \cdot 3003
\]

\[
\text{number of teams} = 216,216
\]

**ANSWER:**

216,216

---

96. Find the values for $x$ and $y$ that make each matrix equation true.

\[
\begin{bmatrix}
y \\
x
\end{bmatrix} = \begin{bmatrix}
4x - 3 \\
y - 2
\end{bmatrix}
\]

*SOLUTION:

Write the matrix as a system of equations.

\[
y = 4x - 3 \quad \text{(Equation 1)}
\]

\[
x = y - 2 \quad \text{(Equation 2)}
\]

Substitute $4x - 3$ for $y$ in Equation 2.

\[
x = y - 2
\]

\[
x = (4x - 3) - 2
\]

\[
x = 4x - 5
\]

\[
-3x = -5
\]

\[
x = \frac{5}{3} \quad \text{or} \quad x = \frac{11}{3}
\]

Substitute $\frac{5}{3}$ for $x$ in Equation 1.

\[
y = 4x - 3
\]

\[
y = 4 \left( \frac{5}{3} \right) - 3
\]

\[
y = \frac{20}{3} - 3
\]

\[
y = \frac{11}{3} \quad \text{or} \quad y = \frac{11}{3}
\]

Therefore, the values for $x$ and $y$ that make the matrix equation true are $\left( \frac{5}{3}, \frac{11}{3} \right)$ or $\left( \frac{2}{3}, \frac{2}{3} \right)$.

**ANSWER:**

\[
\begin{bmatrix}
\frac{5}{3} & \frac{11}{3} \\
\frac{2}{3} & \frac{2}{3}
\end{bmatrix}
\]
1-1 Functions

97. \[
\begin{bmatrix}
3y \\
10
\end{bmatrix} = \begin{bmatrix}
27 + 6x \\
5y
\end{bmatrix}
\]

**SOLUTION:**
Write the matrix as a system of equations.
\[3y = 27 + 6x \quad \text{(Equation 1)} \]
\[10 = 5y \quad \text{(Equation 2)} \]

Solve Equation 2 for \(y\).
\[10 = 5y \]
\[2 = y \]

Substitute 2 for \(x\) in Equation 1.
\[3y = 27 + 6x \]
\[3(2) = 27 + 6x \]
\[6 = 27 + 6x \]
\[6 - 21 = 6x \]
\[-15 = 6x \]
\[-3.5 = x \]

Therefore, the values for \(x\) and \(y\) that make the matrix equation true are \((-3.5, 2)\).

**ANSWER:**
\((-3.5, 2)\)

98. \[
\begin{bmatrix}
9 & 11 \\
3x + 3y & 2x + 1
\end{bmatrix}
\]

**SOLUTION:**
Write the matrix as a system of equations.
\[9 = 3x + 3y \quad \text{(Equation 1)} \]
\[11 = 2x + 1 \quad \text{(Equation 2)} \]

Solve Equation 2 for \(x\).
\[11 = 2x + 1 \\
10 = 2x \\
5 = x \]

Substitute 5 for \(x\) in Equation 1.
\[9 = 3x + 3y \]
\[9 = 3(5) + 3y \]
\[9 = 15 + 3y \]
\[6 = 3y \]
\[-2 = y \]

Therefore, the values for \(x\) and \(y\) that make the matrix equation true are \((5, -2)\).

**ANSWER:**
\((5, -2)\)

99. Use any method to solve the system of equations.
State whether the system is consistent, dependent, independent, or inconsistent.
\[2x + 3y = 36 \]
\[4x + 2y = 48 \]

**SOLUTION:**
Solve each equation for \(y\).
\[2x + 3y = 36 \]
\[3y = 36 - 2x \]
\[y = 12 - \frac{2}{3}x \]
\[4x + 2y = 48 \]
\[2y = 48 - 4x \]
\[y = 24 - 2x \]

Graph each equation to determine the point at which the two lines intersect.

The graphs intersect at the point \((9, 6)\), which is the solution of the system. The system is consistent and independent because the equations have different slopes, the graphs of the equations intersect, and there is one solution.

**ANSWER:**
\((9, 6);\) consistent and independent
1-1 Functions

100. \[5x + y = 25\]
\[10x + 2y = 50\]

**SOLUTION:**
Solve the first equation for \(y\).
\[5x + y = 25\]
\[y = 25 - 5x\]

Substitute \(25 - 5x\) for \(y\) in the second equation.
\[10x + 2y = 50\]
\[10x + 2(25 - 5x) = 50\]
\[10x + 50 - 10x = 50\]
\[50 = 50\]

Because \(50 = 50\) is always true, there are an infinite number of solutions. The graphs of the equations are the same line, as shown below.

Therefore, the system is consistent and dependent.

**ANSWER:**
infinity many solutions; consistent and dependent

101. \[7x + 8y = 30\]
\[7x + 16y = 46\]

**SOLUTION:**
Since one of the variables in each equation has the same coefficient, the elimination method can be used to solve this system.

\[7x + 16y = 46\]
\[- 7x + 8y = 30\]
\[8y = 16\]
\[y = 2\]

Substitute 2 for \(y\) in one of the equations to solve for \(x\).
\[7x + 16(2) = 46\]
\[7x + 32 = 46\]
\[7x = 14\]
\[x = 2\]

The solution of the system is \((2, 2)\). The system is consistent and independent because the equations have different slopes, the graphs of the equations intersect, and there is one solution.

**ANSWER:**
\((2, 2)\); consistent and independent
1-1 Functions

102. BUSINESS A used book store sells 1400 paperback books per week at $2.25 per book. The owner estimates that they will sell 100 fewer books for each $0.25 increase in price. What price will maximize the income of the store?

**SOLUTION:**
Write a quadratic function \( P(x) \) to describe the bookstore's profit as a function of \( x \).

\[
P(x) = (1400 - 100x)(2.25 + 0.25x)
\]

\[
= 3150 + 350x - 225x - 25x^2
\]

\[
= -25x^2 + 125x + 3150
\]

Graph \( P(x) \) to determine the maximum value of the function.

The graph has a maximum of $3306.25 for \( x = 2.5 \). Therefore, to achieve a maximum income of $3306.25 per week, the books should be sold for $2.50 apiece.

**ANSWER:**
$2.50

103. \( A' \)

**SOLUTION:**
\( A' \) represents the complement of set \( A \). To find the complement of set \( A \), identify all elements of \( U \) that are not in set \( A \).

\( A = \{3, 6, 7, 10\} \quad U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \)

So, \( A' = \{1, 2, 4, 5, 8, 9, 11, 12\} \)

**ANSWER:**
\{1, 2, 4, 5, 8, 9, 11, 12\}

104. \( A \cup B \)

**SOLUTION:**
\( A \cap B \) represents the union of set \( A \) and set \( B \). The union of \( A \) and \( B \) is the set of all elements that belong to set \( A \), set \( B \), or both sets.

\( A = \{3, 6, 7, 10\} \quad B = \{2, 6, 9\} \)

So, \( A \cup B = \{2, 3, 6, 7, 9, 10\} \).

**ANSWER:**
\{2, 3, 6, 7, 9, 10\}

105. \( B \cap C \)

**SOLUTION:**
\( B \cap C \) represents the intersection of set \( B \) and set \( C \). The intersection of \( B \) and \( C \) is the set of all elements found in both \( B \) and \( C \).

\( B = \{2, 6, 9\} \quad C = \{1, 11, 12\} \)

Because there are no elements that belong to both \( B \) and \( C \), the intersection of \( B \) and \( C \) is the empty set. So, \( B \cap C = \emptyset \).

**ANSWER:**
\( \emptyset \)
1-1 Functions

106. \( A \cap B \)

\textbf{SOLUTION:} \\
\( A \cap B \) represents the intersection of set \( A \) and set \( B \). The intersection of \( A \) and \( B \) is the set of all elements found in both \( A \) and \( B \).

\[ A = \{3, 6, 7, 10\} \quad B = \{2, 6, 9\} \]

So, \( A \cap B = \{6\} \).

\textbf{ANSWER:} \\
\{6\}

107. SAT/ACT A circular cone with a base of radius 5 has been cut as shown in the figure. What is the height of the smaller top cone?

\[ h_a^2 + 5^2 = 13^2 \]
\[ h_a^2 = 13^2 - 5^2 \]
\[ h_a^2 = 144 \]
\[ h_a = 12 \]

Since two angles of Triangle \( A \) are congruent to two angles of Triangle \( B \), Triangle \( A \) and Triangle \( B \) are similar triangles, and \( \frac{h_a}{13} = \frac{h_b}{8} \).

\[ \frac{12}{13} = \frac{h_b}{8} \]
\[ \frac{96}{13} = h_b \]

Therefore, the correct answer choice is B.

\textbf{ANSWER:} \\
B
1-1 Functions

108. **REVIEW** Which function is linear?
   
   - F \( f(x) = x^2 \)
   - G \( g(x) = 2.7 \)
   - H \( f(x) = \sqrt{9 - x^2} \)
   - J \( g(x) = \sqrt{x - 1} \)

   **SOLUTION:**
   
   A linear function is any function whose graph is a straight line. The function in choice A is quadratic and will have a U-shaped graph. The function in choice B is a constant function and will have a graph that is a horizontal line. The functions in choices C and D are square root functions, which will have nonlinear graphs. Therefore, the only function with a graph that is a straight line is \( g(x) = 2.7 \), so the correct answer choice is G.

   **ANSWER:**
   
   G

109. Louis is flying from Denver to Dallas for a convention. He can park his car in the Denver airport long-term lot or in the nearby shuttle parking facility. The long-term lot costs $1 per hour or any fraction thereof with a maximum charge of $6 per day. In the shuttle facility, he has to pay $4 for each day or part of a day. Which parking lot is less expensive if Louis returns after 2 days and 3 hours?
   
   - A shuttle facility
   - B airport lot
   - C They will both cost the same.
   - D Cannot be determined with the information given.

   **SOLUTION:**
   
   The long-term lot costs $1 per hour, with a maximum charge of $6 per day. So, for 2 days and 3 hours the long-term lot will cost \( 2(6) + 3(1) \) or $15.

   The shuttle facility costs $4 per day. So, for 2 days and 3 hours, the charge will be for 3 days, and the shuttle facility will cost \( 4(3) \) or $12. Therefore, the shuttle facility is less expensive, and the correct answer choice is A.

   **ANSWER:**
   
   A

110. **REVIEW** Given \( y = 2.24x + 16.45 \), which statement best describes the effect of moving the graph down two units?
   
   - F The \( y \)-intercept increases.
   - G The \( x \)-intercept remains the same.
   - H The \( x \)-intercept increases.
   - J The \( y \)-intercept remains the same.

   **SOLUTION:**
   
   When the graph of \( y = 2.24x + 16.45 \) is shifted two units down, the equation that corresponds to the transformed graph is \( y = 2.24x + 16.45 - 2 \) or \( y = 2.24x + 14.45 \).

   Set \( y = 2.24x + 16.45 \) equal to 0 to find the root of the equation, which corresponds to the \( x \)-intercept of the graph:

   \[
   2.24x + 16.45 = 0 \\
   2.24x = -16.45 \\
   x \approx -7.34
   \]

   Set \( y = 2.24x + 14.45 \) equal to 0.

   \[
   2.24x + 14.45 = 0 \\
   2.24x = -14.45 \\
   x \approx -6.45
   \]

   Because \(-7.34 < -6.45\), the \( x \)-intercept increased as a result of the transformation, and the correct answer choice is H.

   **ANSWER:**
   
   H