Find \((f + g)(x), (f - g)(x), (f \cdot g)(x),\) and \(\left(\frac{f}{g}\right)(x)\) for each \(f(x)\) and \(g(x)\). Indicate any restrictions in domain or range.

14. \(f(x) = -x^2 + 6\)
\(g(x) = 2x^2 + 3x - 5\)

\[\text{SOLUTION:}\]
\[
(f + g)(x) = f(x) + g(x) \\
= -x^2 + 3x + 1
\]
\[
(f - g)(x) = f(x) - g(x) \\
= -x^2 + 6 - (2x^2 + 3x - 5) \\
= -3x^2 - 3x + 11
\]
\[
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (-x^2 + 6)(2x^2 + 3x - 5) \\
= -2x^4 - 3x^3 + 17x^2 + 18x - 30
\]
\[
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 \\
= \frac{-x^2 + 6}{2x^2 + 3x - 5}, 2x^2 + 3x - 5 \neq 0 \\
= \frac{-x^2 + 6}{2x^2 + 3x - 5}, x \neq 1 \text{ or } -\frac{5}{2}
\]

16. POPULATION In a particular county, the population of the two largest cities can be modeled by \(f(x) = 200x + 25\) and \(g(x) = 175x - 15\), where \(x\) is the number of years since 2000 and the population is in thousands.

a. What is the population of the two cities combined after any number of years?

b. What is the difference in the populations of the two cities?

\[\text{SOLUTION:}\]
a. The population of the cities after \(x\) years is the sum of the individual populations.
\[
(f + g)(x) = f(x) + g(x) \\
= 200x + 25 + 175x - 15 \\
= 375x + 10
\]
b. The difference in the populations of the cities is given by:
\[
(f - g)(x) = 200x + 25 - (175x - 15) \\
= 25x + 40
\]
6-1 Operations on Functions

36. **FINANCE** A ceramics store manufactures and sells coffee mugs. The revenue \( r(x) \) from the sale of \( x \) coffee mugs is given by \( r(x) = 6.5x \). Suppose the function for the cost of manufacturing \( x \) coffee mugs is \( c(x) = 0.75x + 1850 \).

a. Write the profit function.

b. Find the profit on 500, 1000, and 5000 coffee mugs.

**SOLUTION:**
a. The profit function \( P(x) \) is given by 
\[
P(x) = r(x) - c(x)
\]
where \( r(x) \) is the revenue function and \( c(x) \) is the cost function. 
So:
\[
P(x) = 6.5x - (0.75x + 1850)
= 5.75x - 1850
\]

b. 
\[
P(500) = 5.75(500) - 1850
= 1025
\]
\[
P(1000) = 5.75(1000) - 1850
= 5750 - 1850
= 3900
\]
\[
P(5000) = 5.75(5000) - 1850
= 26,900
\]

Perform each operation if \( f(x) = x^2 + x - 12 \) and 
\( g(x) = x - 3 \). State the domain of the resulting function.

38. \((f - g)(x)\)

**SOLUTION:**
\[
(f - g)(x) = f(x) - g(x)
= x^2 + x - 12 - (x - 3)
= x^2 - 9
\]
\(D = \{\text{all real numbers}\}\)

39. \(2(g \cdot f)(x)\)

**SOLUTION:**
\[
2(g \cdot f)(x) = 2 \cdot g(x) \cdot f(x)
= 2(x - 3)(x^2 + x - 12)
= 2x^3 - 4x^2 - 30x + 72
\]
\(D = \{\text{all real numbers}\}\)

40. \(\left(\frac{f}{g}\right)(x)\)

**SOLUTION:**
\[
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}
= \frac{x^2 + x - 12}{x - 3}, x \neq 3
= \frac{(x + 4)(x - 3)}{(x - 3)}, x \neq 3
= x + 4, x \neq 3
\]
\(D = \{x | x \neq 3\}\)

If \( f(x) = x + 2 \), \( g(x) = -4x + 3 \), and \( h(x) = x^2 - 2x + 1 \), find each value.

53. \[([f + g] \cdot h)(1)\]

**SOLUTION:**
\[
([f + g] \cdot h)(x) = [f + g](x) \cdot h(x)
= [f(x) + g(x)] \cdot h(x)
= f(x) \cdot h(x) + g(x) \cdot h(x)
\]

Substitute \( x = 1 \).
\[
([f + g] \cdot h)(1) = f(1) \cdot h(1) + g(1) \cdot h(1)
= (3)(0) + (-1)(0)
= 0
\]
61. **CHALLENGE** Given \( f(x) = \sqrt{x^3} \) and \( g(x) = \sqrt{x^6} \) determine the domain for each of the following.

a. \( g(x) \cdot g(x) \)

b. \( f(x) \cdot f(x) \)

**SOLUTION:**

a. \( g(x) \cdot g(x) = \sqrt{x^6} \cdot \sqrt{x^6} \)
\[
= x^6
\]
\( D = \{ \text{all real numbers} \} \)

b. \( f(x) \cdot f(x) = \sqrt{x^3} \cdot \sqrt{x^3} \)
\[
= x^3
\]
Since \( f(x) \) is defined for \( x \geq 0 \), the domain of \( f(x) \cdot f(x) \) is \( \{ x | x \geq 0 \} \).